FREAK WAVES AND STOCHASTIC WAVE GROUPS

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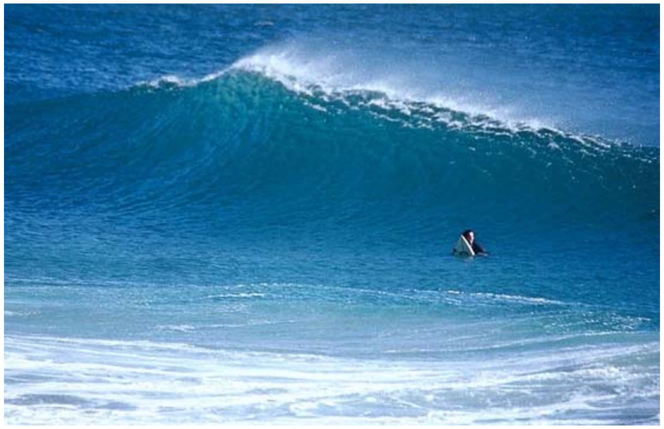
A NATURAL BEAUTY !



Freak waves

Rogue waves

Giant waves

Extreme waves

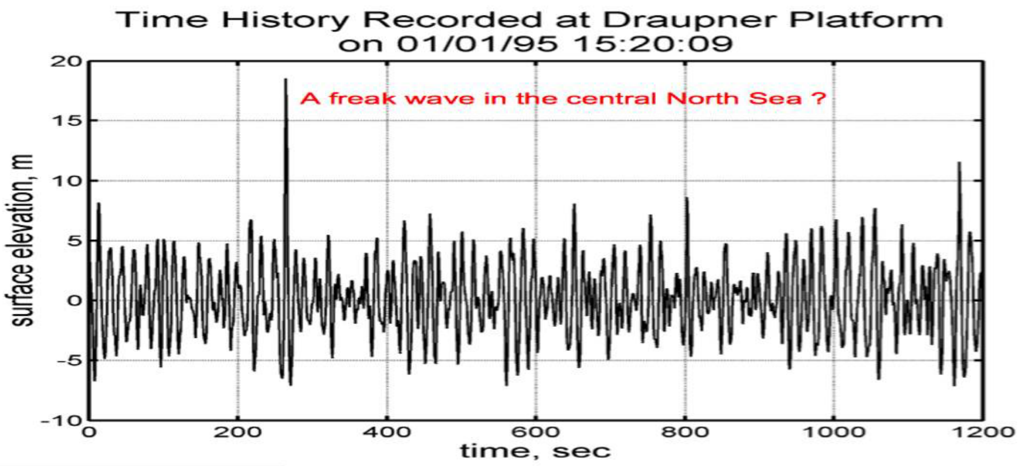
Rogue waves

Extreme waves

Giant waves

Freak waves

DRAUPNER EVENT JANUARY 1995



H_{max}=25.6 m

with

$H_{max}/H_s>2$!

1 in 200,000 waves

according to Rayleigh law

CAN WE MODEL AND PREDICT EXTREME WAVES ?

GAUSSIAN THEORY ⟷ NONLINEAR WAVES

↓

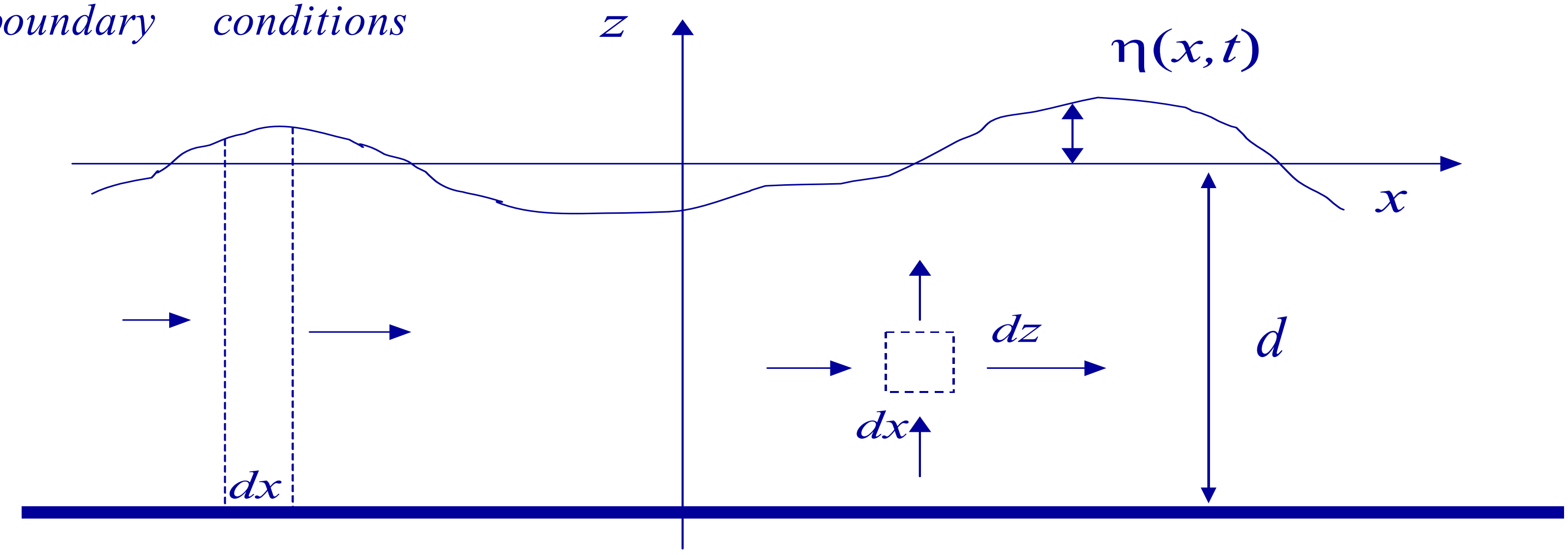
STOCHASTIC WAVE GROUPS

STOKES EQUATIONS

$$\begin{cases} \dfrac{\partial^2\Phi}{\partial x^2}+\dfrac{\partial^2\Phi}{\partial z^2}=0 \\[2ex] \left(\dfrac{\partial\Phi}{\partial z}\right)_{z=\eta}=\dfrac{\partial\eta}{\partial t}+\dfrac{\partial\eta}{\partial x}\left(\dfrac{\partial\Phi}{\partial x}\right)_{z=\eta} \\[2ex] \left(\dfrac{\partial\Phi}{\partial t}\right)_{z=\eta}+\dfrac{1}{2}\left[\left(\dfrac{\partial\Phi}{\partial x}\right)^2+\left(\dfrac{\partial\Phi}{\partial z}\right)^2\right]_{z=\eta}+g\eta=f(t) \\[2ex] \text{boundary conditions} \end{cases}$$

$$v_z=\frac{\partial\Phi}{\partial z} \qquad v_x=\frac{\partial\Phi}{\partial x}$$

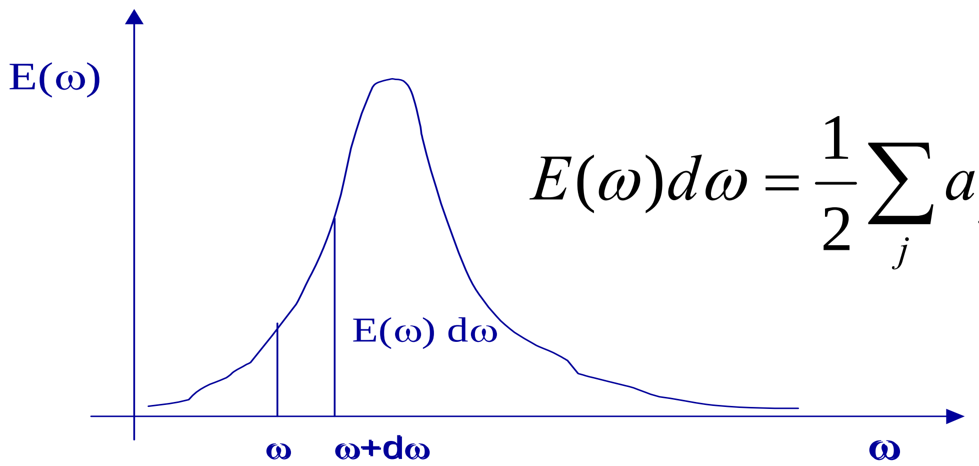
Inviscid, irrotational



LINEAR WAVES : GAUSSIAN SEAS

$$\eta(x,t) = \sum_{j=1}^{N} a_j \cos\left(k_j x + \omega_j t + \varepsilon_j\right)$$

$$\varepsilon_j \quad \textit{uniformly random in } [0, 2\pi]$$



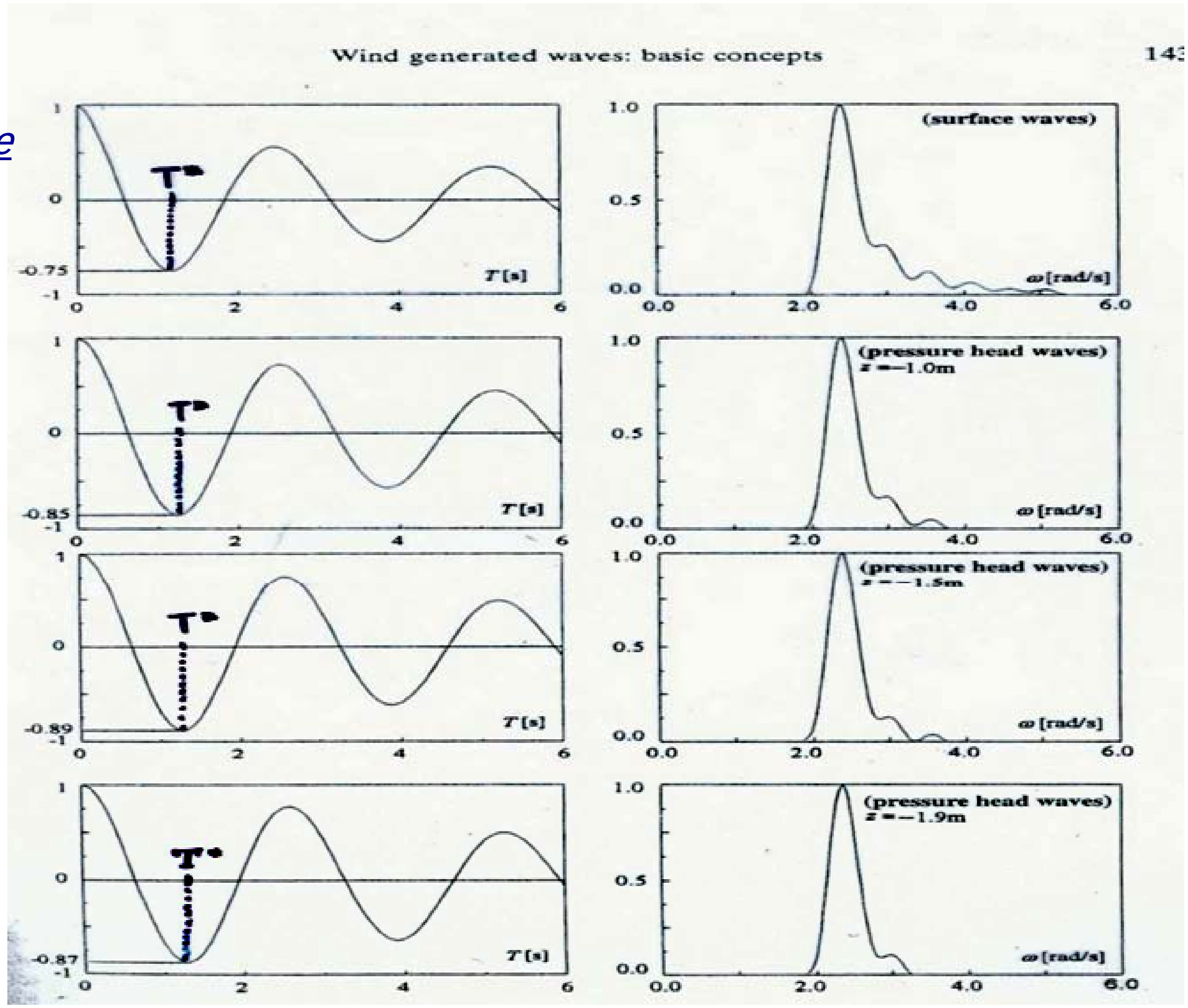
$$E(\omega)d\omega = \frac{1}{2}\sum_j a_j^2 \qquad \omega < \omega_j < \omega + d\omega$$

$$\psi(T) = \langle \eta(x_0, t_0)\eta(x_0, t_0 + T)\rangle = \int_0^{\infty} E(\omega)\cos\omega T \, d\omega$$

Stationarity Ergodicity Gaussianity

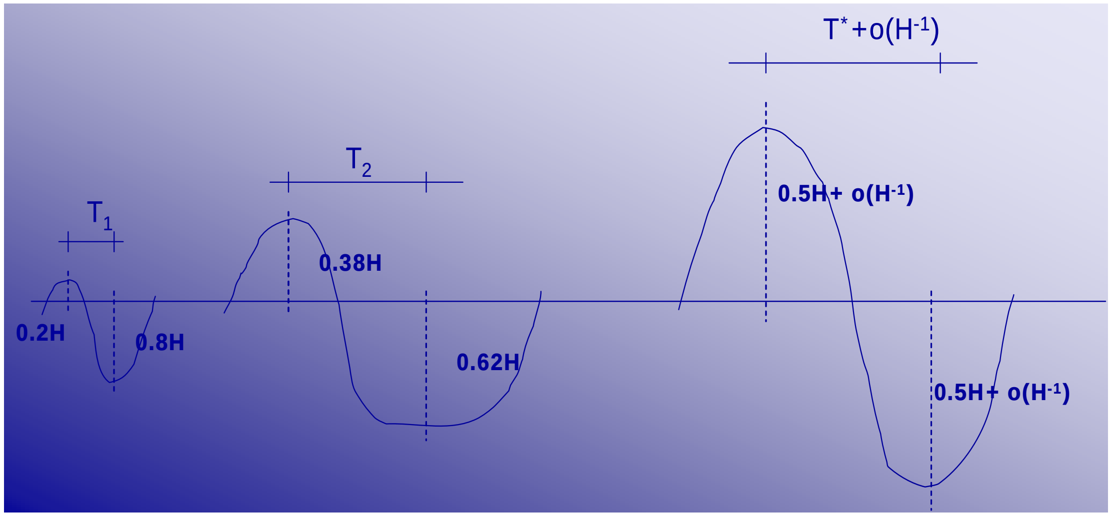
TYPICAL WAVE SPECTRA OF THE MEDITERRANEAN SEA*

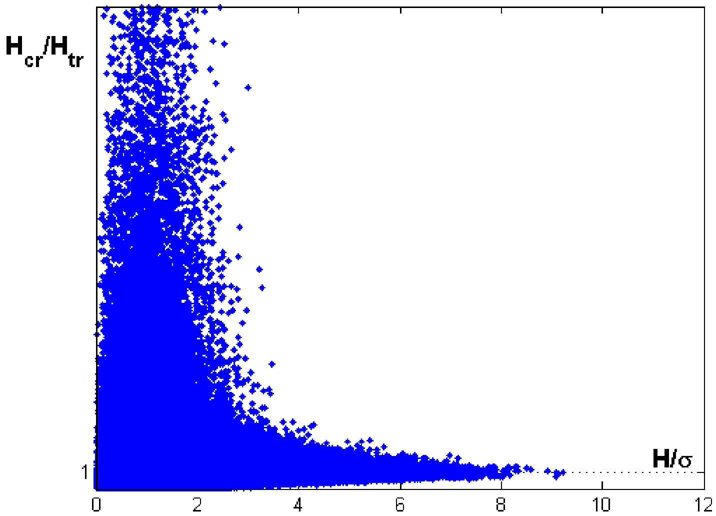
Time covariance *Spectrum*

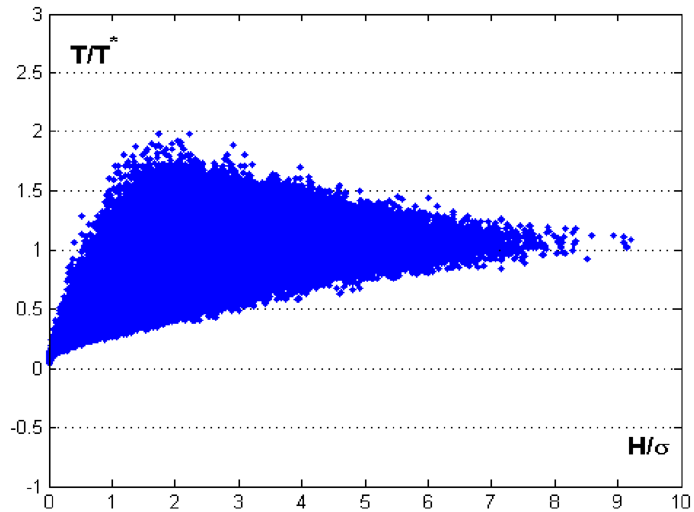


*from *Boccotti P.* **Wave Mechanics** *2000 Elsevier*

OCCURRENCE OF A HIGH WAVE IN GAUSSIAN SEAS*







*Theory of quasi-determinism, *Boccotti P.* **Wave Mechanics** *2000 Elsevier*

EXPECTED SHAPE OF THE SEA LOCALLY TO A HIGH WAVE*

What happens in the neighborhood of a point $\mathbf{x}_0$ if a large crest followed by large trough are recorded in time at $\mathbf{x}_0$?

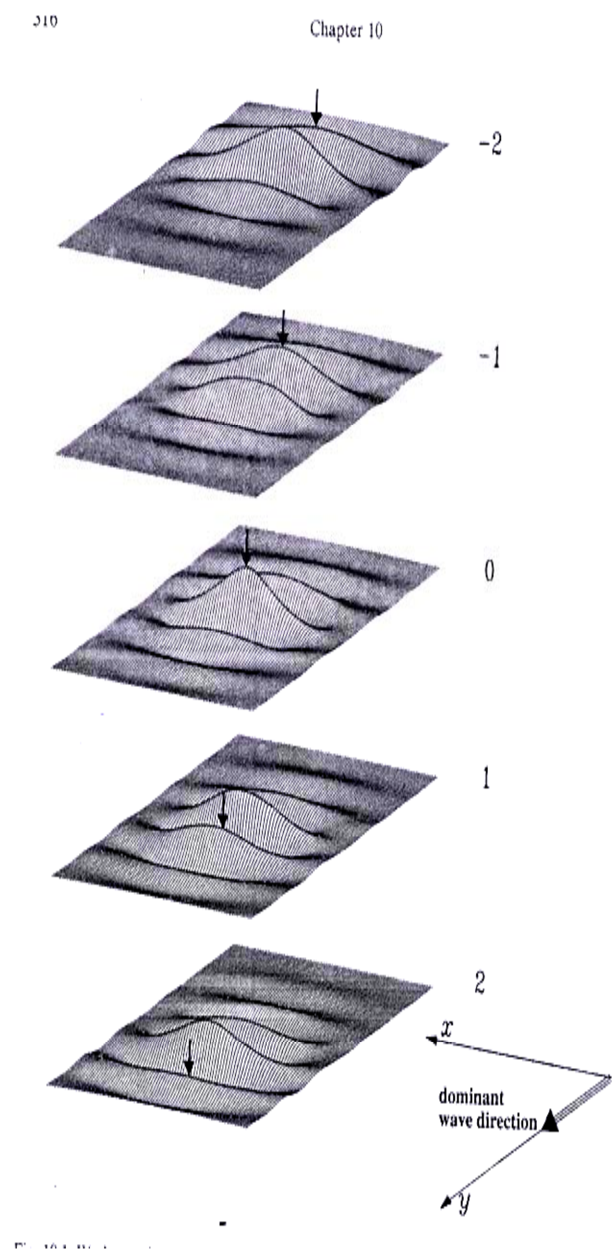
$$\Pr\left[\begin{array}{l}\eta(\mathrm{x}_0+\mathrm{X},t_0+T)\in(u,u+du)\\ \\ conditione\ d\quad to\\ \eta(\mathrm{x}_0,t_0)=H/2,\ \ \eta(\mathrm{x}_0,t_0+T_2^*)=-H/2\end{array}\right]$$

$$\Downarrow\quad \frac{H}{\sigma}\rightarrow\infty$$

$$\eta_c\left(\mathrm{x}_0+\mathrm{X},t_0+T\right)=\frac{H}{2}\Psi(X,T)$$

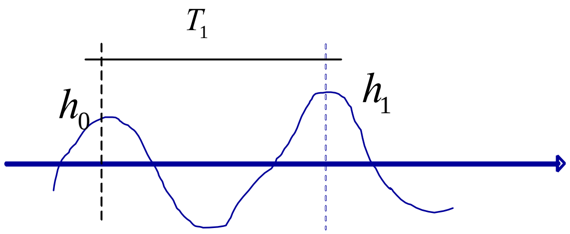
SPACE-TIME covariance

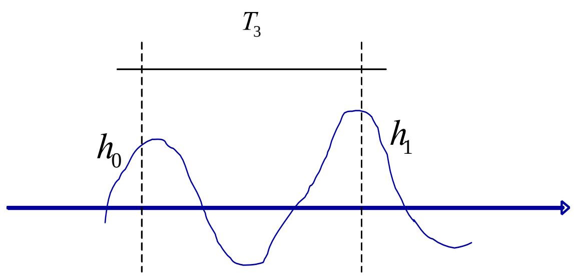
$$\Psi(\mathrm{X},T)=\left\langle\eta(x_0,t_0)\eta(x_0+\mathrm{X},t_0+T)\right\rangle$$

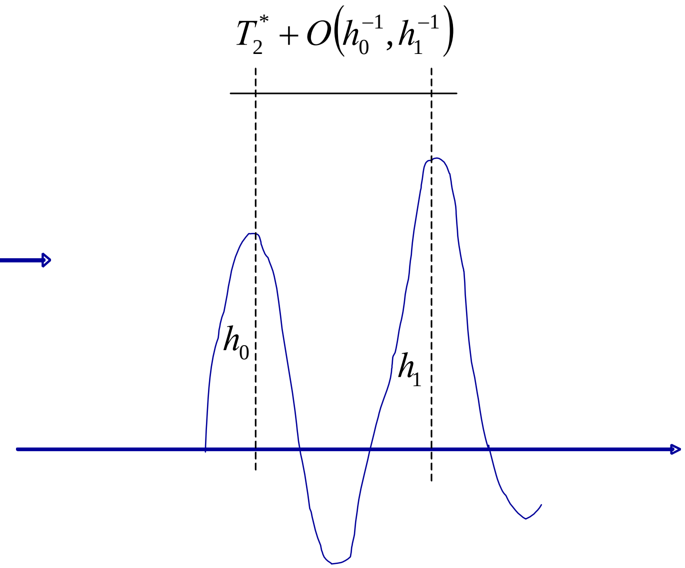


* *Boccotti P.* **Wave Mechanics** *2000 Elsevier*

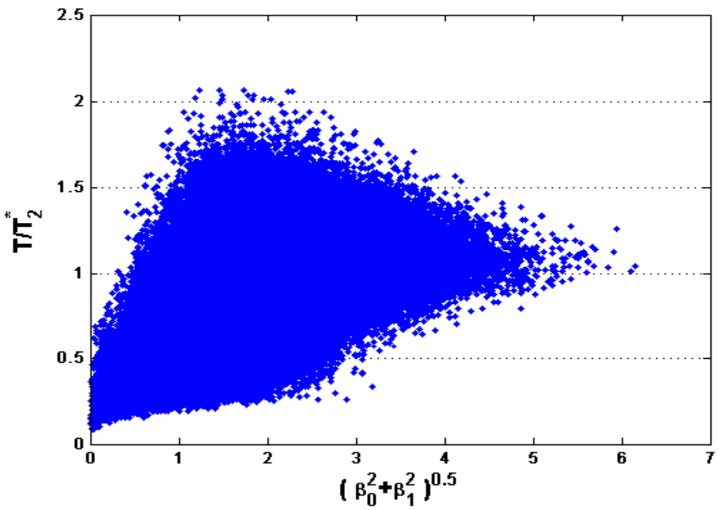
SUCCESSIVE WAVE CRESTS IN TIME*

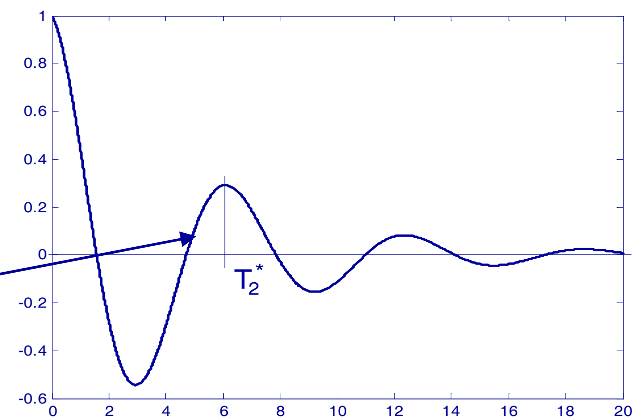






$$as \quad h_0 \to \infty, \quad h_1 \to \infty$$





$$p(\beta_0,\beta_1) = \frac{\beta_0\beta_1}{1-k^2}\exp\left[-\frac{\beta_0^2+\beta_1^2}{2(1-k^2)}\right] I_0\left(\frac{k\beta_0\beta_1}{1+k^2}\right)$$

Bivariate Rayleigh

* Fedele F., **Successive wave crests in a Gaussian sea**, *Probabilistic Eng. Mechanics* 2005 vol. 20, Issue 4, 355-363

EXPECTED SHAPE OF THE SEA LOCALLY TO TWO SUCCESSIVE WAVE CRESTS *

What happens in the neighborhood of a point $\mathbf{x}_0$
if two large successive wave crests are recorded in time at $\mathbf{x}_0$?

$$\Pr\begin{bmatrix} \eta(\mathrm{x}_0+\mathrm{X},t_0+T)\in(u,u+du) \\ \\ conditioned\ \ to \quad \eta(\mathrm{x}_0,t_0)=h_1,\ \ \eta(\mathrm{x}_0,t_0+T_2^*)=h_2 \end{bmatrix}$$

$$\Downarrow \quad \frac{h_1}{\sigma}\to\infty, \quad \frac{h_2}{\sigma}\to\infty$$

$$\eta_c(\mathbf{X},T)=\frac{\psi(0)h_1-h_2\psi(T_2^*)}{\psi^2(0)-\psi(T_2^*)^2}\ \Psi(\mathbf{X},T)+\frac{\psi(0)h_2-h_1\psi(T_2^*)}{\psi^2(0)-\psi(T_2^*)^2}\ \Psi(\mathbf{X},T-T_2^*)$$

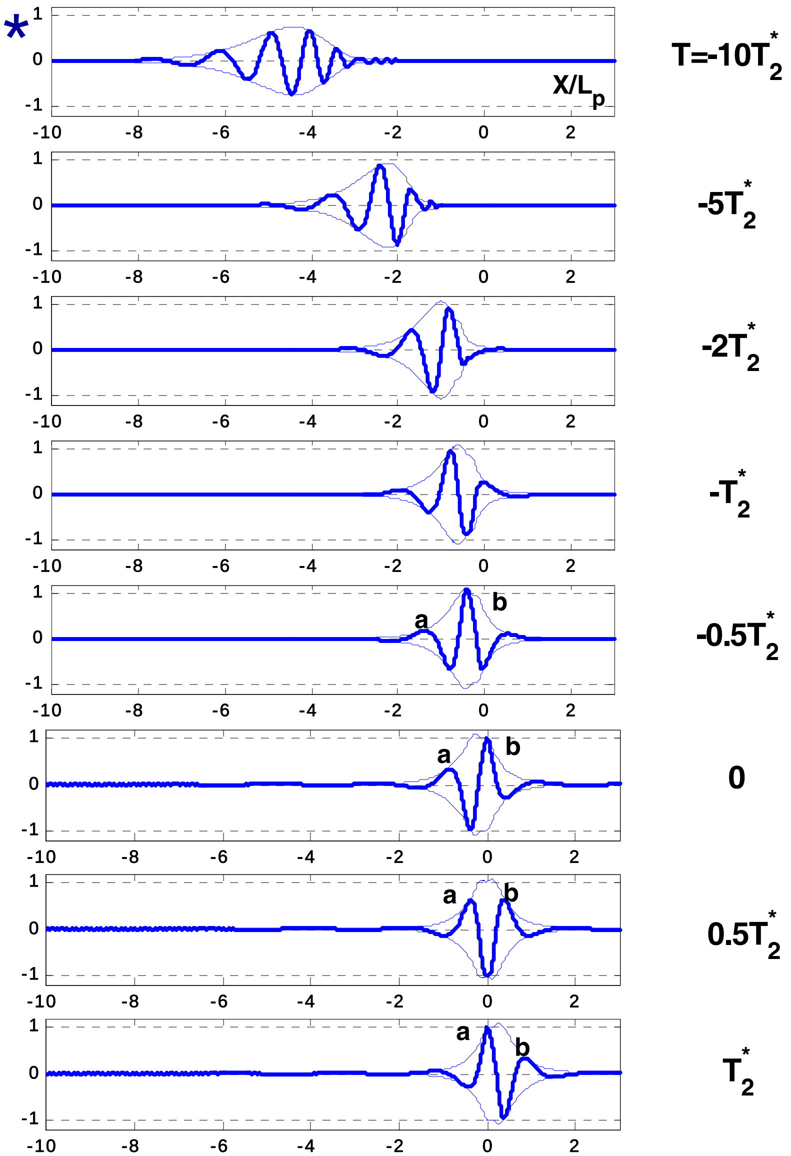
What is hidden "behind" this equation ?

* Fedele F., 2006. **On wave groups in a Gaussian sea.** *Ocean Engineering 2006 (in press)*

A SINGLE WAVE GROUP CAUSES TWO SUCCESSIVE WAVE CRESTS !*

$$\eta_{ex}(X,T) = h\ \Psi(X,T)$$

SPACE-TIME covariance



<u>STOCHASTIC WAVE GROUP</u>

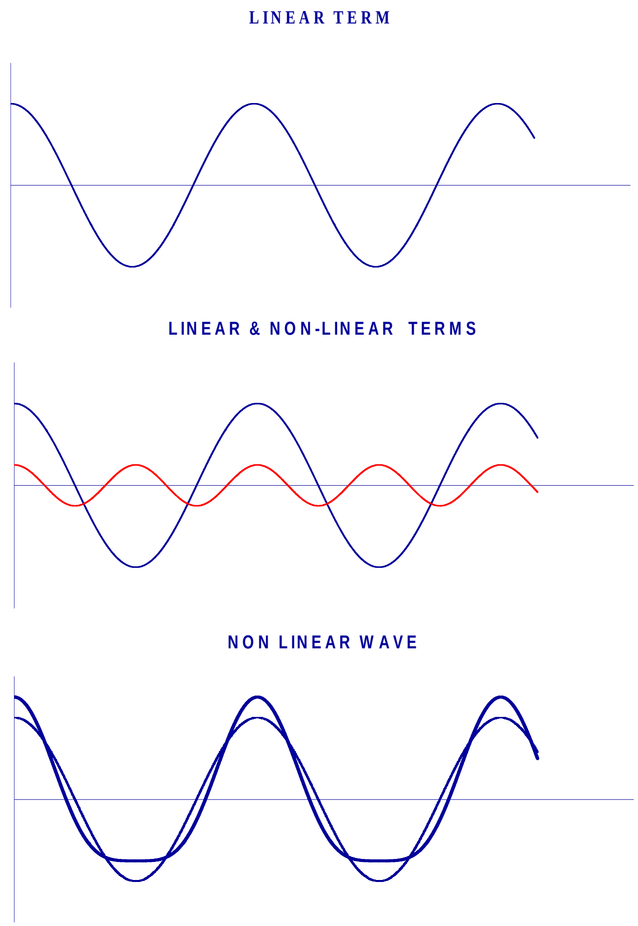
amplitude h random variable distributed according to Rayeligh

stochastic family of wave groups

* Fedele F., 2006. **On wave groups in a Gaussian sea.** *Ocean Engineering 2006 (in press)*

NONLINEAR RANDOM SEAS

Second order effects: **BOUND WAVES**

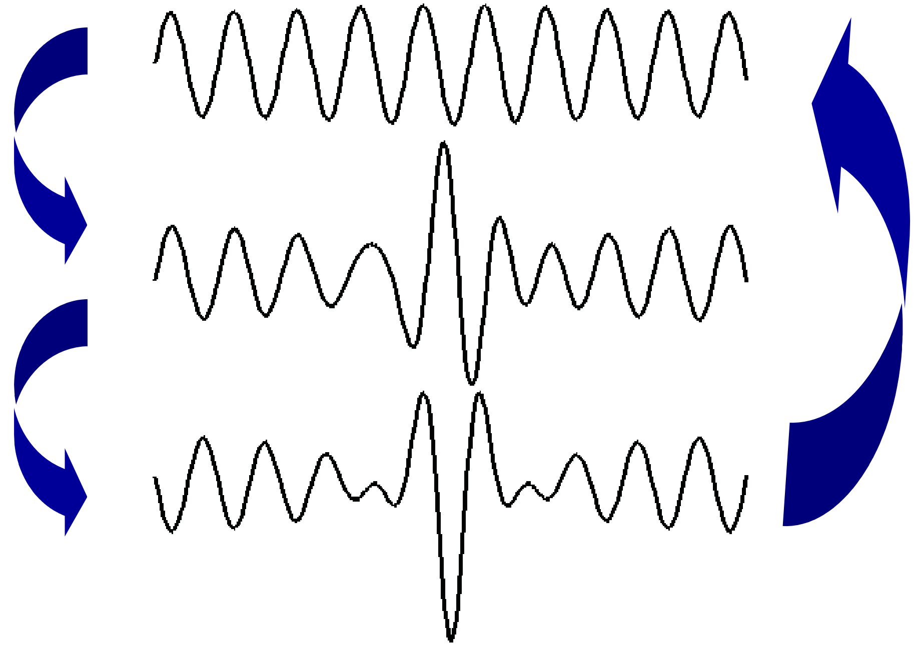


Crest–trough asymmetry : **skewness>0**

Wave height almost linear

<u>Effects on Short time scale</u> : wave period

Third order effects : **FOUR-WAVE RESONANCE**



Crest-trough symmetry : **kurtosis>3**

Benjamin-Feir Index BFI=steepness/bandwitdh

Modulation instability: Fermi Ulam-Pasta recurrence

<u>Effects on slow time scale</u> : wave period/steepness^2

DOMINANT ONLY IN UNIDIRECTIONAL NARROW-BAND SEAS !

NONLINEAR RANDOM SEAS cont'd

$$\tilde{\eta}(\underline{\mathbf{x}},t)=\eta(\underline{\mathbf{x}},t)+\eta_2(\underline{\mathbf{x}},t)$$

Third order effects :
FOUR-WAVE RESONANCE
(*WEAK WAVE TURBULENCE***)**

$$\eta(\underline{\mathbf{x}},t)=\frac{1}{\pi}\sum_n\sqrt{\frac{\omega_n}{2g}}\left|B_n(t)\right|\cos\left(\underline{\mathbf{k}}_n\cdot\underline{\mathbf{x}}+\left|\varphi_n(t)\right|\right)$$

$$\frac{dB_n}{dt}+i\omega_nB_n=-i\sum_{p,q,r}T_{npqr}\delta_{n+p-q-r}B_p^*B_qB_r$$

Conserved quantities :
Hamiltonian
Wave action
Wave momentum

$$\mathrm{H}=\sum_n\omega_nB_n(t)B_n^*(t)+\frac{1}{2}\sum_{n,p,q,r}T_{npqr}\delta_{n+p-q-r}B_n^*(t)B_p^*(t)B_q(t)B_r(t)$$

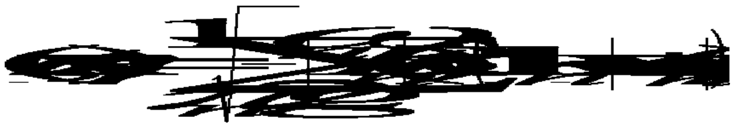
$$\mathbf{A}=\sum_nB_n(t)B_n^*(t)\qquad\mathbf{M}=\sum_n\mathbf{k}_nB_n(t)B_n^*(t)$$

Second order effects:
BOUND WAVES

$$\eta_2(\underline{\mathbf{x}},t)=\sum_{p,q}\sqrt{\omega_p\omega_q}\left|B_p(t)\right|\left|B_q(t)\right|\Gamma_{pq}^{+}\cos\left(\left(\underline{\mathbf{k}}_p+\underline{\mathbf{k}}_q\right)\cdot\underline{\mathbf{x}}+\varphi_p+\varphi_q\right)+$$

$$+\sum_{p,q}\sqrt{\omega_p\omega_q}\left|B_p(t)\right|\left|B_q(t)\right|\Gamma_{pq}^{-}\cos\left(\left(\underline{\mathbf{k}}_p-\underline{\mathbf{k}}_q\right)\cdot\underline{\mathbf{x}}+\varphi_p-\varphi_q\right)$$

NONLINEAR EVOLUTION OF A STOCHASTIC WAVE GROUP*



At (x=0, t=0) we impose that all the elementary waves are in phase (focussing)

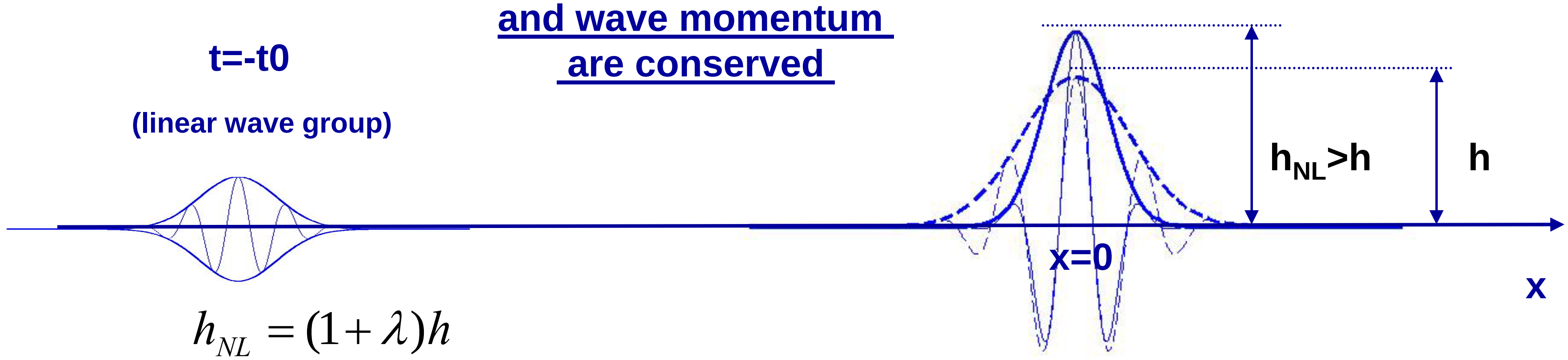
$$\varphi_n(0) = 0 \qquad n = 1,....N$$

t=0

Hamiltonian , wave action and wave momentum are conserved

t=-t0

(linear wave group)



$$h_{NL} = (1+\lambda)h$$

λ **Self-focussing parameter**

α **Skewness parameter**

$$\tilde{h}_{nl} = (1+\lambda)h + \alpha(1+\lambda)^2 h^2$$

h Rayleigh distributed

$$\Pr(H_{\max} > h) = \exp\left[-\frac{(1+\lambda)^2}{8\alpha^2}\left(1-\sqrt{1+\frac{4\alpha}{(1+\lambda)^2}\frac{h}{\sigma}}\right)\right]$$

***Fedele F**. 2006. **Extreme Events in nonlinear random seas**. J. of Offshore Mechanics and Arctic Eng., ASME, **128**, 11-16.

$$\xi_{nl} = \xi \sqrt[4]{1 + \chi \xi^2}$$

Symmetric
Third order effects

$$\downarrow$$

$$\tilde{\xi}_{nl} = \xi_{nl} + \frac{\mu}{2} \xi_{nl}^2$$

Second order
Bound effects

$$\downarrow$$

$$\Pr[\tilde{\xi}_{nl} > y] = \exp\left(-\frac{1}{2} \xi^2 (y) \right)$$

$$\tilde{\xi}_{nl} = y$$

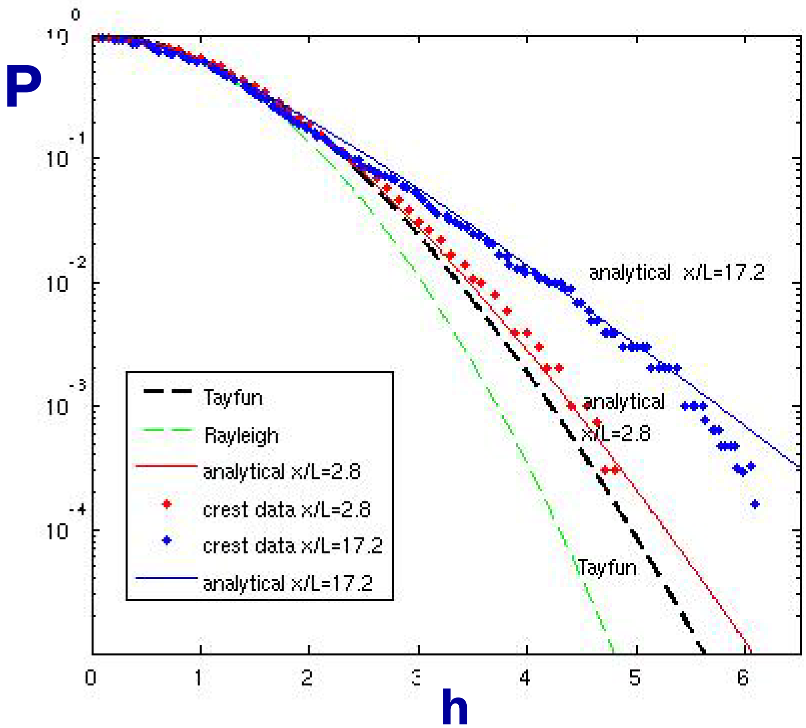
$$\chi = (4 \ln 2) BFI^2 \left(1 - \frac{\cos\left(\frac{1}{2} \arctan \frac{\varepsilon^2 \omega_p T}{BFI^2 \, 2 \ln 2} \right)}{\sqrt[4]{1 + \left(\frac{\varepsilon^2 \omega_p T}{BFI^2 \, 2 \ln 2} \right)^2}} \right)$$

$$BFI = \frac{\sqrt{2}\varepsilon}{\Delta k / k_0}$$

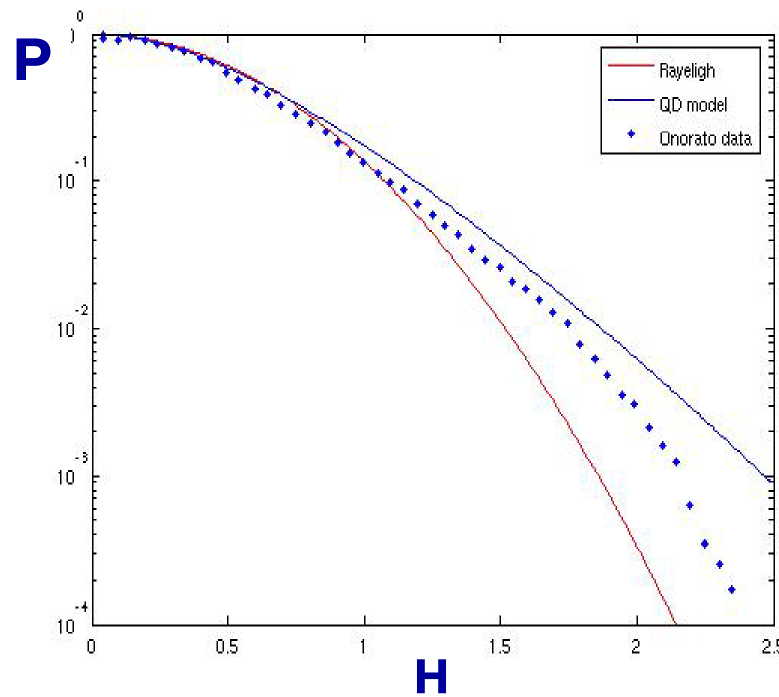
THE PROBABILITY OF EXCEEDING A FIXED WAVE AMPLITUDE

Wave tank experiments:
<u>unidirectional narrow-band seas</u> (Onorato et all. 2005)

THIRD ORDER MODULATION + SECOND ORDER EFFECTS



Wave crest



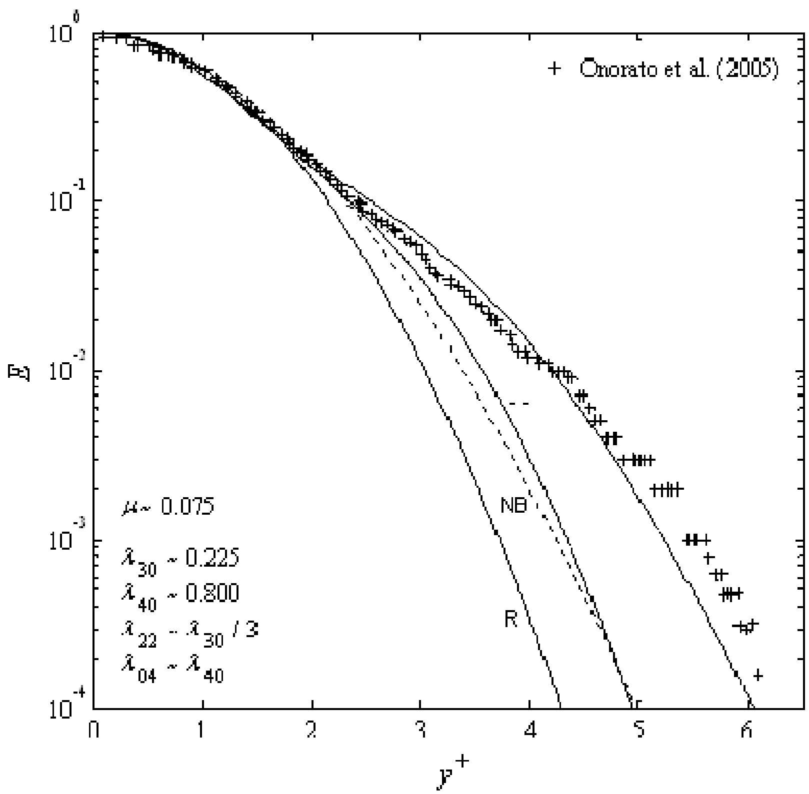
Wave height

THE PROBABILITY OF EXCEEDANCE cont'd

Wave tank experiments:
unidirectional narrow-band seas (Onorato et all. 2005)

unrealistic ocean conditions

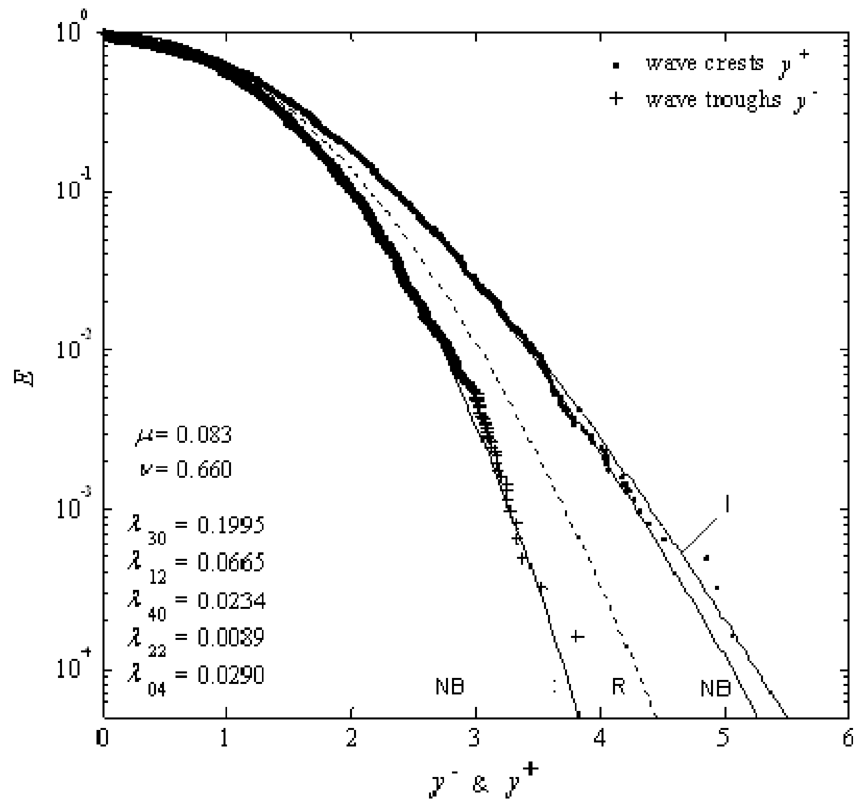
MODULATION + SECOND ORDER EFFECTS



TERN platform, time series (6,000 waves)

realistic ocean conditions

SECOND ORDER EFFECTS DOMINANT



*Tayfun A. Fedele F., **Wave heights and nonlinear effects**.Ocean Engineering (submitted)

Fedele F., Tayfun A. **Explaining extreme waves by a theory of Stochastic wave groups. PROCEEDINGS of OMAE 2006 (in press)