THE OCCURRENCE OF EXTREME CRESTS AND THE NONLINEAR INTERACTION IN RANDOM SEAS

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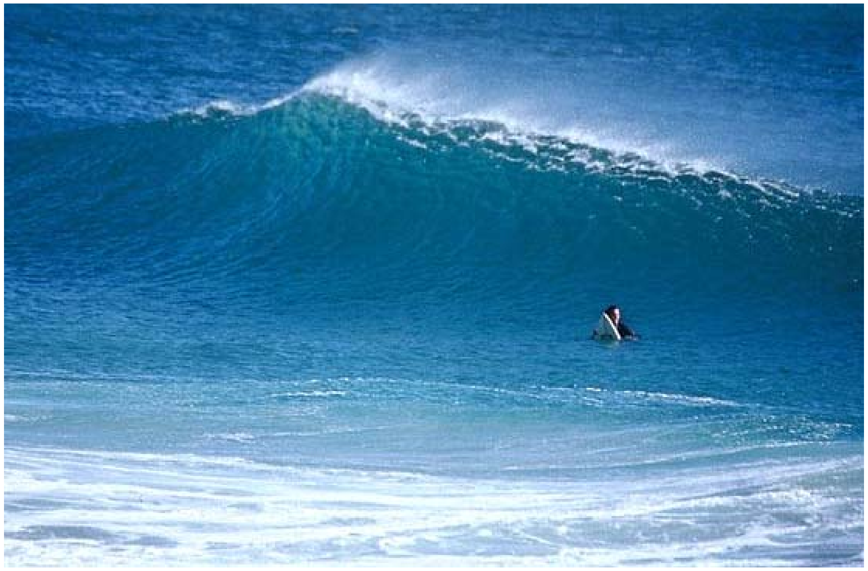


THE ZAKHAROV EQUATION

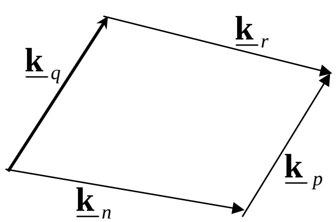
$$\eta(\underline{\mathbf{x}},t)=\frac{1}{\pi}\sum_n\sqrt{\frac{\omega_n}{2g}}\,B_n(t)\exp\left(\underline{\mathbf{k}}_n\cdot\underline{\mathbf{x}}+\omega_n t\right)+c.c.$$

$$\frac{dB_n}{dt}+i\omega_n B_n=-i\sum_{p,q,r}T_{npqr}\,\delta_{n+p-q-r}\,B_p^* B_q B_r$$

Conserved quantities : Hamiltonian , wave action and momentum

$$\mathbf{H}=\sum_n\omega_n B_n(t)B_n^*(t)+\frac{1}{2}\sum_{n,p,q,r}T_{npqr}\,\delta_{n+p-q-r}\,B_n^*(t)B_p^*(t)B_q(t)B_r(t)$$

$$\mathbf{A}=\sum_n B_n(t)B_n^*(t)$$



Quartet interaction

$$\underline{\mathbf{k}}_n+\underline{\mathbf{k}}_p=\underline{\mathbf{k}}_q+\underline{\mathbf{k}}_r$$

$$\mathbf{M}=\sum_n\mathbf{k}_n B_n(t)B_n^*(t)$$

SUFFICIENT CONDITIONS TO HAVE AN EXTREME CREST

$$\eta(\underline{\mathbf{x}},t)=\frac{1}{\pi}\sum_n\sqrt{\frac{\omega_n}{2g}}\left|B_n(t)\right|\cos\left(\underline{\mathbf{k}}_n\cdot\underline{\mathbf{x}}+\omega_n t+\left|\varphi_n(t)\right|\right)$$

$$B_n(t)=\left|B_n(t)\right|\exp\left[i\varphi_n(t)\right]$$

Set initial conditions ⇒ $B_n(t=-t_0)=\tilde{B}_n\exp(i\tilde{\varphi}_n)$

At (x=0, t=0) we impose that all the harmonic components are in phase (focusing)

$$\varphi_n(0)=0 \qquad n=1,\ldots.N$$

⇓

From the ZAKHAROV EQUATION

$$\nabla\eta=\mathbf{0}\quad and\quad \frac{\partial\eta}{\partial t}=0\quad at\quad (\underline{\mathbf{x}}=\mathbf{0},t=0)$$

Stationarity at (x=0,t=0)

Amplitude at (x=0,t=0)

$$H_{\max}=\frac{1}{\pi}\sum_n\sqrt{\frac{\omega_n}{2g}}\left|B_n(0)\right|$$

SUFFICIENT CONDITIONS TO HAVE AN EXTREME CREST

Maximum amplitude at (x=0,t=0)

$$H_{\max} = \frac{1}{\pi}\sum_n \sqrt{\frac{\omega_n}{2g}}\left|B_n(0)\right|$$

Optimization problem

$$\max \frac{1}{\pi}\sum_n \sqrt{\frac{\omega_n}{2g}}\left|B_n(0)\right|$$

with the following constraints

$$\sum_n \omega_n \left|B_n(0)\right|^2 + \frac{1}{2}\sum_{n,p,q,r} T_{npqr}\delta_{n+p-q-r}\left|B_n(0)\right|\left|B_p(0)\right|\left|B_q(0)\right|\left|B_r(0)\right|$$

$$= \sum_n \omega_n \tilde{B}_n^{\,2} + \frac{1}{2}\sum_{n,p,q,r} T_{npqr}\delta_{n+p-q-r}\tilde{B}_n\tilde{B}_p\tilde{B}_q\tilde{B}_r$$

$$\sum_n \left|B_n(0)\right|^2 = \sum_n \tilde{B}_n^{\,2} \qquad \sum_n \mathbf{k}_n \left|B_n(0)\right|^2 = \sum_n \mathbf{k}_n \tilde{B}_n^{\,2}$$

HOW TO CHOOSE THE INITIAL CONDITIONS

Theory of Quasi-Determinism of Boccotti

$$\eta_{\det}(\underline{\mathbf{x}},t)=\frac{H}{\sigma^2}\int E(\underline{\mathbf{k}})\cos\left(\underline{\mathbf{k}}_n\cdot\underline{\mathbf{x}}-\omega_n t\right)d\underline{\mathbf{k}} \qquad \frac{H}{\sigma}\to\infty$$

$$\frac{N_{cr}(b,T)}{N_+(b,T)}\to 1 \qquad if\ \frac{H}{\sigma}\to\infty$$

$$\Pr\left[H>b\right]=\frac{N_+(b,T)}{N_+(0,T)}=\exp\left(-\frac{b^2}{2\sigma^2}\right) \qquad if\ \frac{b}{\sigma}\to\infty$$

Discrete form $$\eta_{\det}(\underline{\mathbf{x}},t)=\frac{H}{\sigma^2}\sum_n\frac{1}{2}a_n^2\cos\left(\underline{\mathbf{k}}_n\cdot\underline{\mathbf{x}}-\omega_n t\right) \qquad \frac{H}{\sigma}\to\infty$$

Initial conditions which give the highest crest at (x=0,t=0) for linear waves

$$\tilde{B}_n=\frac{\pi H}{2\sigma^2\sqrt{\omega_n/2g}}a_n^2 \qquad \tilde{\varphi}_n=0 \qquad n=1,....N$$

THE CONSTRAINED OPTIMIZATION PROBLEM

$$\max_{(X_1,\ldots X_N)\in\Re^N} \sum_n w_n X_n \qquad X_n \geq 0$$

$$\sum_n X_n^2 = \sum_n \tilde{X}_n^{\,2} \qquad \sum_n \mathbf{k}_n X_n^2 = \sum_n \mathbf{k}_n \tilde{X}_n^{\,2}$$

$$\sum_n w_n X_n^2 + \varepsilon^2 \sum_{n,p,q,r} T_{npqr}\delta_{n+p-q-r} X_n X_p X_q X_r = \sum_n w_n \tilde{X}_n^2 + \varepsilon^2 \sum_{n,p,q,r} T_{npqr}\delta_{n+p-q-r} \tilde{X}_n \tilde{X}_p \tilde{X}_q \tilde{X}_r$$

$$H_{\max} = (1+\lambda)H \qquad \frac{H}{\sigma} \to \infty \qquad \lambda = \frac{1}{\pi}\sum \sqrt{w_n} X_n - 1$$

THE EXTREME CREST AMPLITUDE

Third order effects due to nonlinear interaction of free harmonics

$$H_{\max} = (1+\lambda)H \qquad \frac{H}{\sigma} \to \infty \qquad \lambda = \frac{1}{\pi}\sum \sqrt{w_n} X_n - 1$$

Second order effects due to bound harmonics

$$h = \sum{}_n A_n + \frac{1}{4}\sum{}_{n,s} \Gamma_{ns} A_n A_s$$

$$H_{\max} = (1+\lambda)H + \alpha k_d H^2 \qquad \alpha = \frac{1}{4\pi^2}\sum{}_{n,s} \Gamma_{ns}\sqrt{w_n w_s} X_n X_s$$

$$\Pr\left(H_{\max} > h\right) = \exp\left[-\frac{(1+\lambda)^2}{8\varepsilon^2\alpha^2}\left(1-\sqrt{1+\frac{4\varepsilon\alpha}{(1+\lambda)^2}\frac{h}{\sigma}}\right)\right]$$

NARROW-BAND SPECTRA

Benjamin-Feir instability

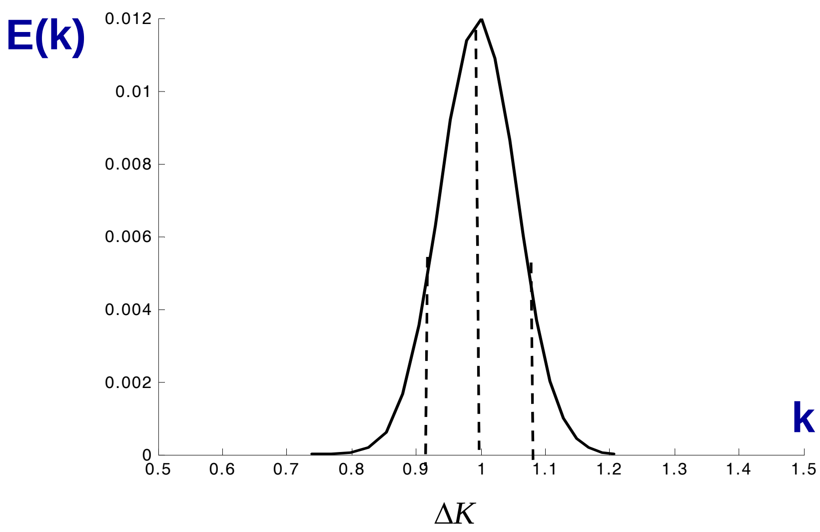
$$\frac{\Delta K}{k} \leq 2\sqrt{2}\,\varepsilon$$

Benjamin-Feir index

$$BFI = \frac{2\sqrt{2}\,\varepsilon}{\Delta K / k}$$

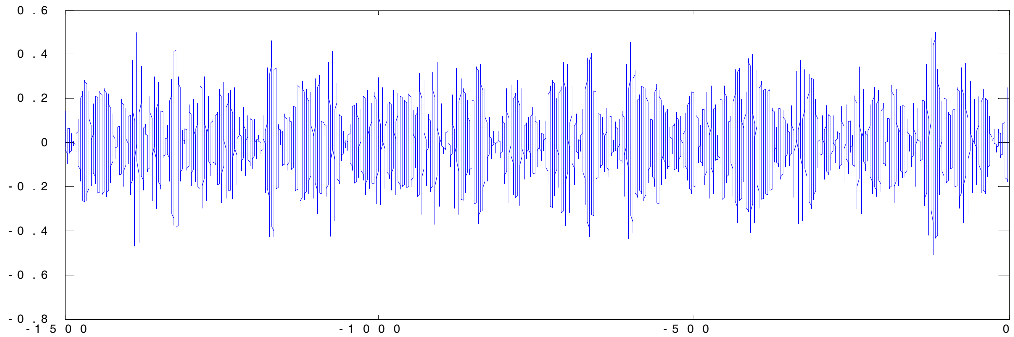
Initial spectrum

$$E(k) = \frac{1}{\sqrt{2\pi\sigma_k^2}} \exp\left[-\frac{(k-1)^2}{2\sigma_k^2}\right]$$

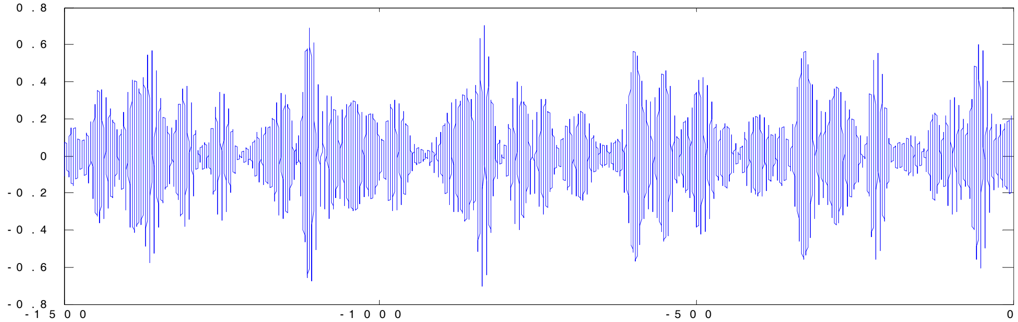


TIME SERIES FROM NUMERICAL SIMULATIONS

BFI=0.9



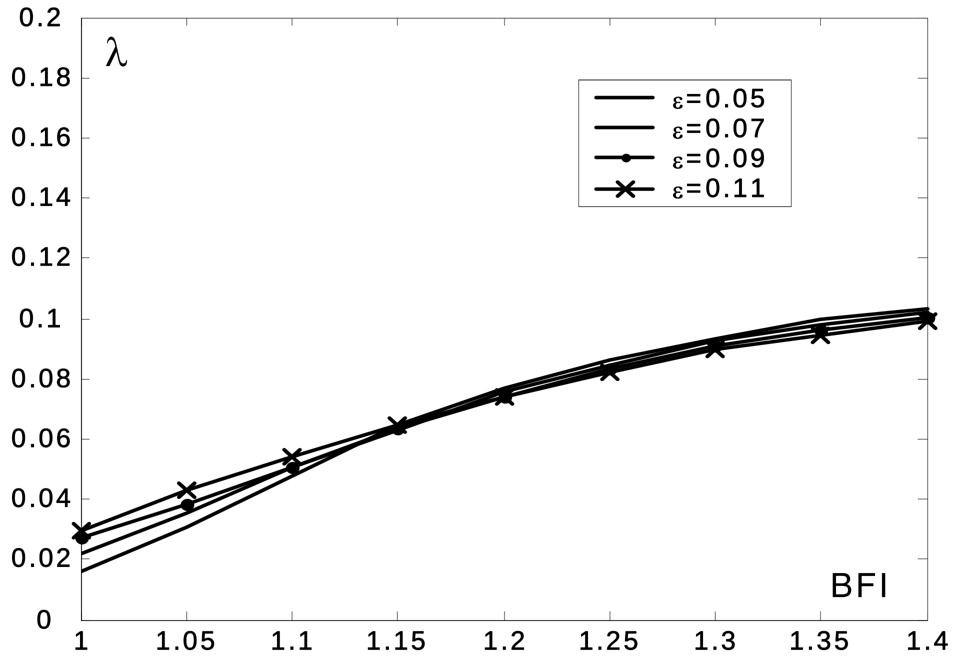
BFI=1.4



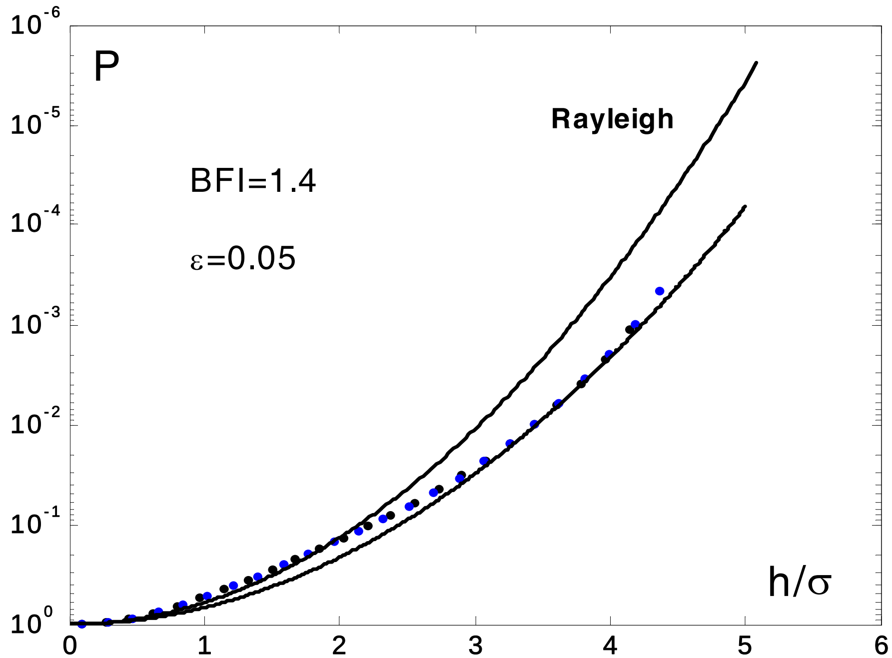
A NEW ANALYTICAL EXPRESSION FOR THE PROBABILITY OF EXCEEDANCE OF A WAVE CREST

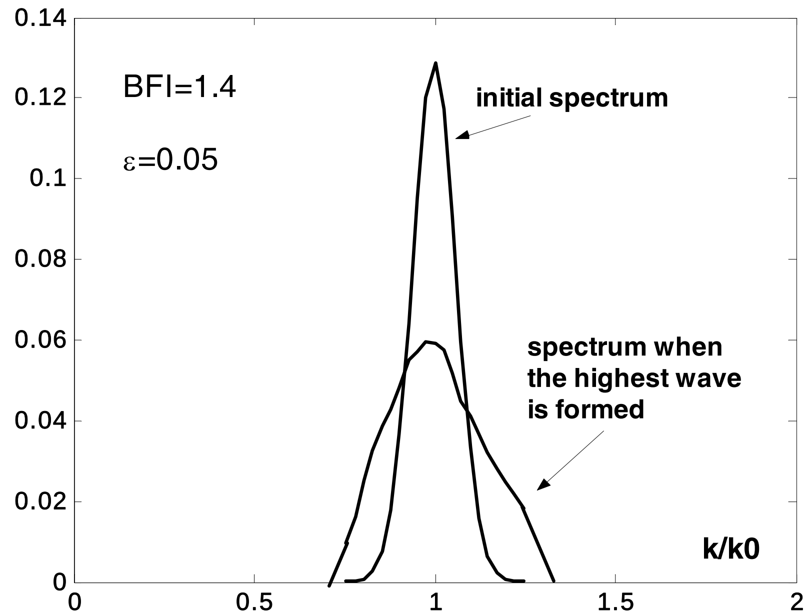
$$\Pr\left(H_{\max} > h\right) = \exp\left[-\frac{h^2}{2(1+\lambda)^2\sigma^2}\right]$$

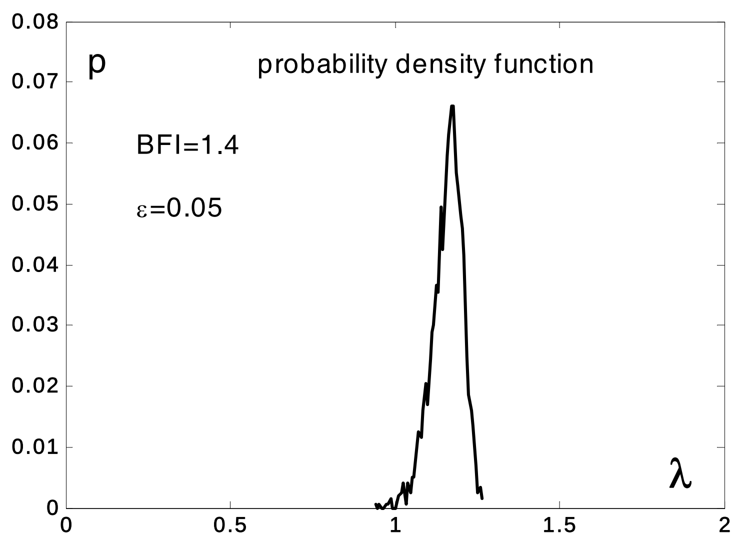
$$\Pr\left(H_{\max} > h\right) = \exp\left[-\frac{(1+\lambda)^2}{8\varepsilon^2\alpha^2}\left(1-\sqrt{1+\frac{4\varepsilon\alpha}{(1+\lambda)^2}\frac{h}{\sigma}}\right)\right]$$

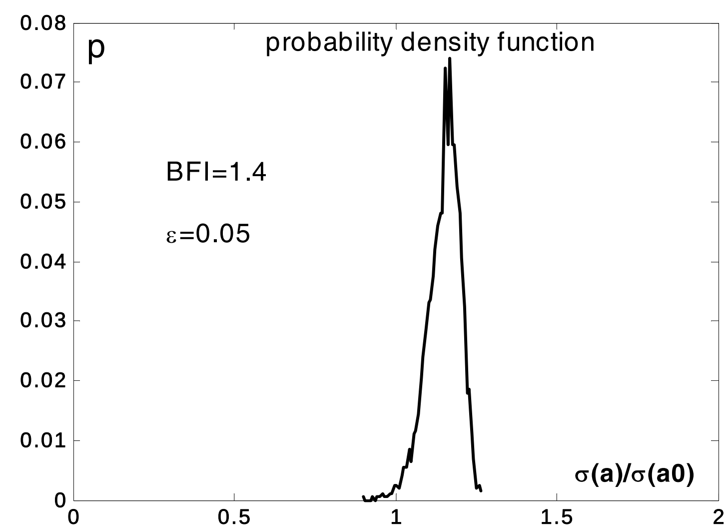


Benjamin-Feir index BFI=1.4

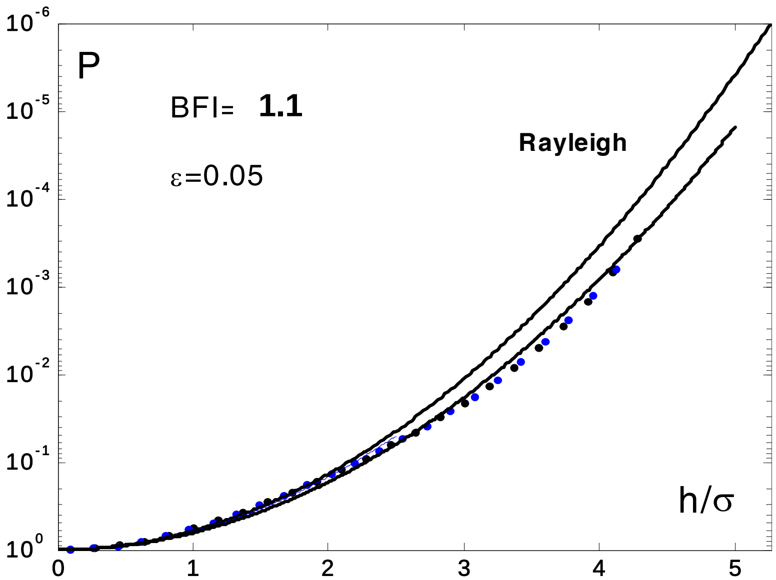


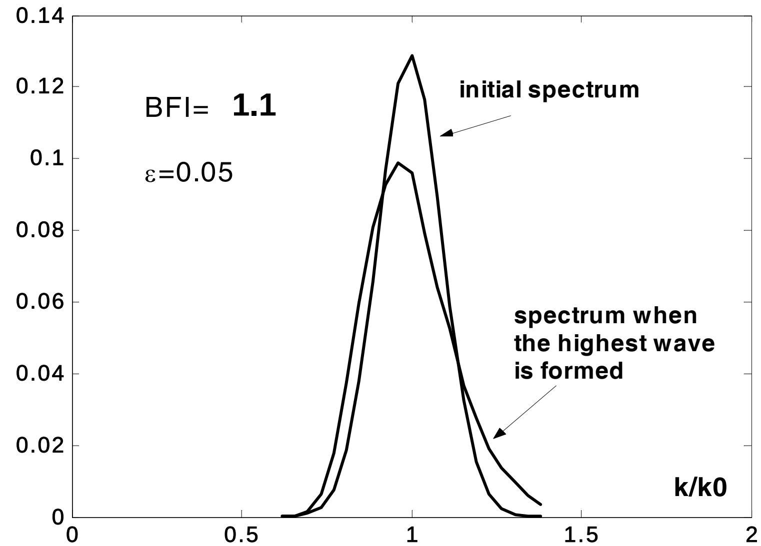


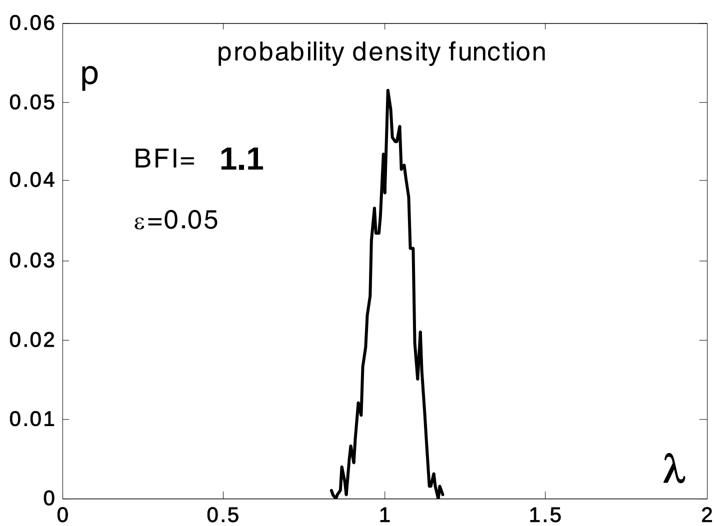


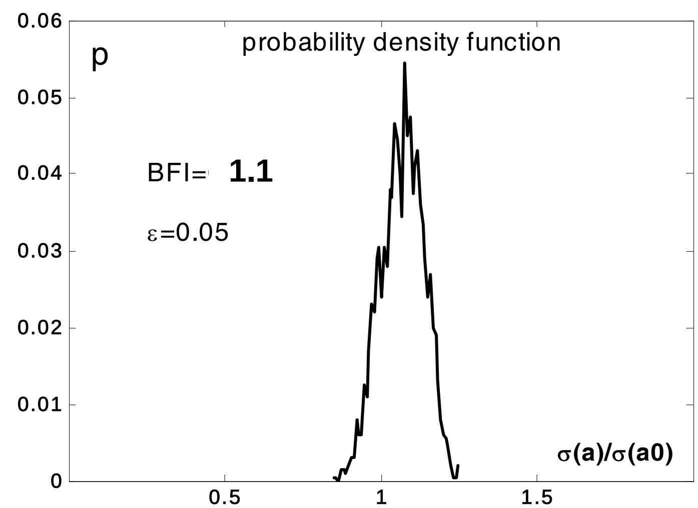


Benjamin-Feir index BFI=1.1





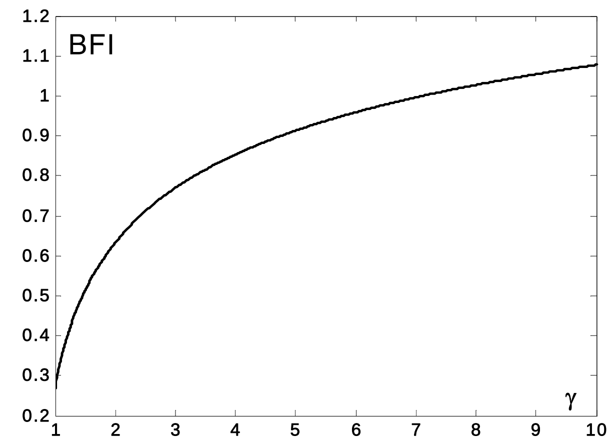




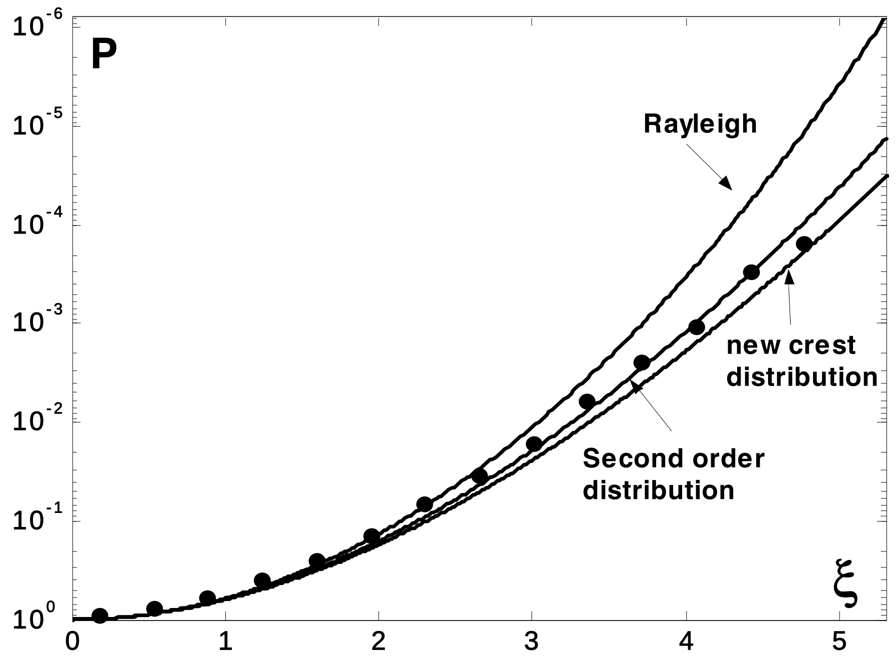
JONSWAP SPECTRA AND DRAUPNER DATA

$$E(k) = \frac{H_s^2}{16\pi} \frac{1}{1 + (k-1)^2 / \delta^2}$$

$$\delta = \sqrt{\frac{8\chi^2}{24\chi^2 + \ln\gamma}}$$



$$\Pr(H_{\max} > h) = \exp\left[-\frac{(1+\lambda)^2}{8\varepsilon^2\alpha^2} \left(1 - \sqrt{1 + \frac{4\varepsilon\alpha}{(1+\lambda)^2} \frac{h}{\sigma}} \right) \right]$$



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LONGUET-HIGGINS
1952 On the statistical distribution of the heights of sea waves, J. Mar. Res.;11:245-266.

LINDGREN
1970 Some properties of a normal process near a local maximum. *Ann. Math. Statist.* 41 1870-1883.
1971 Extreme values of stationary normal processes. *Z. Wahrscheinlichkeitstheorie und Verw. Gebiete* 17 39-47
1972 Local maxima of Gaussian fields. *Ark. Mat.* 10, 195-218.

BOCCOTTI
1978 The distribution of intervals between zeros of random functions of time. (Italian)
Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (8) 65
1978 The statistical properties of waves generated by wind. (Italian) *Atti Accad. Ligure Sci. Lett.* 35, 159--185
1981 On the highest waves in a stationary Gaussian process. Atti Acc. Ligure di Scienze e Lettere;38:271-302.
1982 On the highest sea waves. Monograph Institute of Hydraulics University of Genoa :1-161.
1982 On ocean waves with high crests. Meccanica ;17:16-19.
1983 Some new results on statistical properties of wind waves. Applied Ocean Research;5:134-140.
1989 On mechanics of irregular gravity waves. Atti Acc. Naz. Lincei, Memorie;VIII: 111-170.
1993 A field experiment on the mechanics of irregular gravity waves. J. Fluid Mech. ;252:173-186
1997 A general theory of three-dimensional wave groups. Ocean Engng.;24:265-300.
2000 Wave mechanics for ocean engineering. Elsevier Science:1-496.

TROMANS et ALL
1991 A new model for the kinematics of large ocean waves – application as a design wave –
Shell International Research 1991;publ. 1042.

SUN
1993 Tail probabilities of the maxima of gaussian random fields. The Annals of Probability;21(1):34-71.

REVISITING THE STABILITY OF PULSATILE PIPE FLOW

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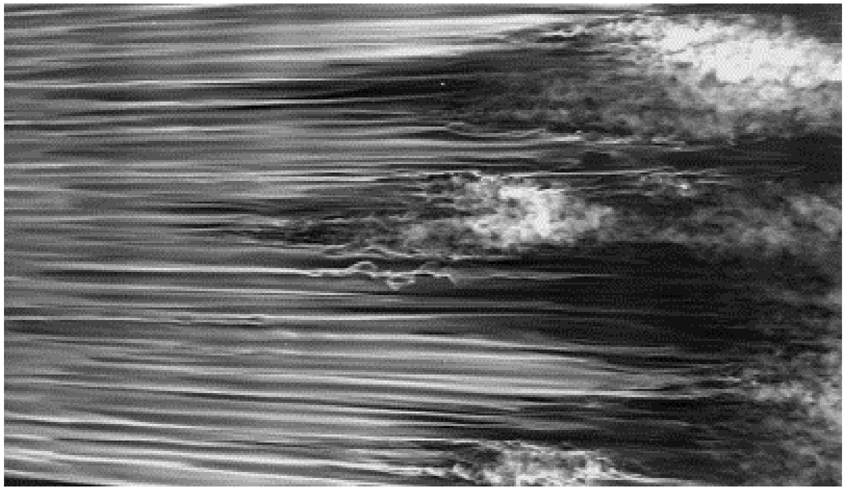
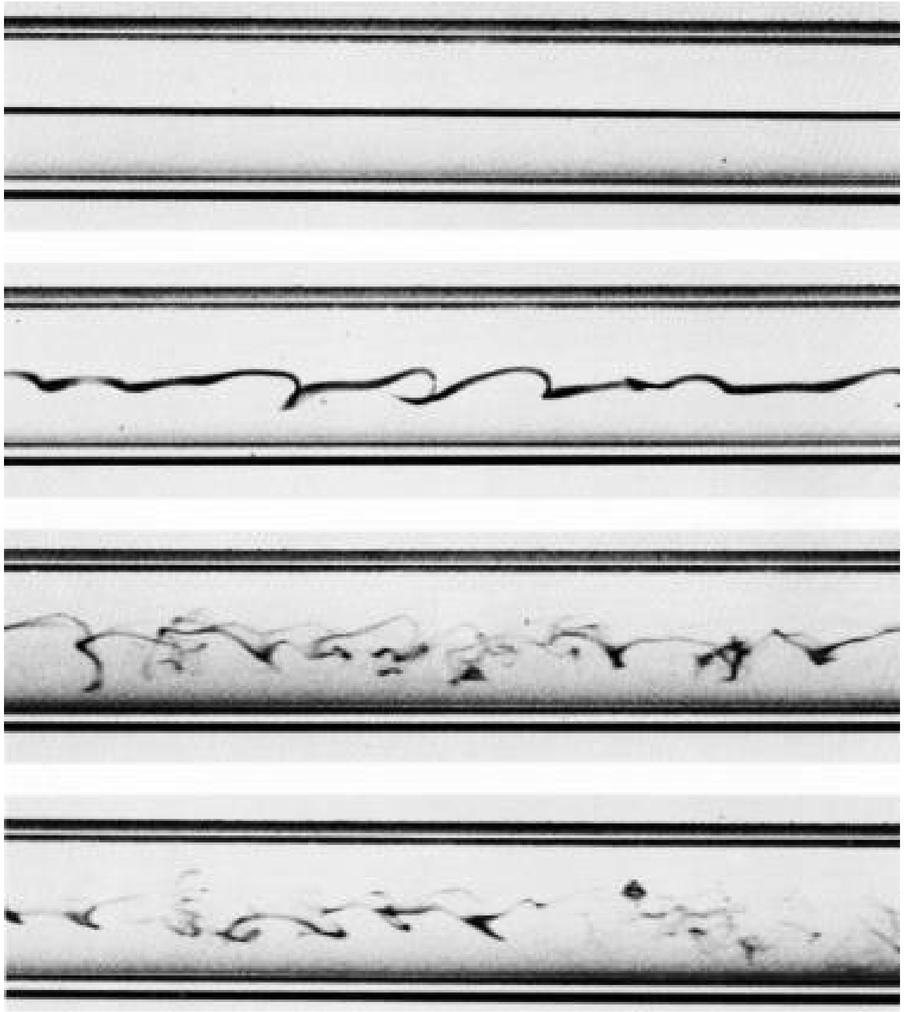
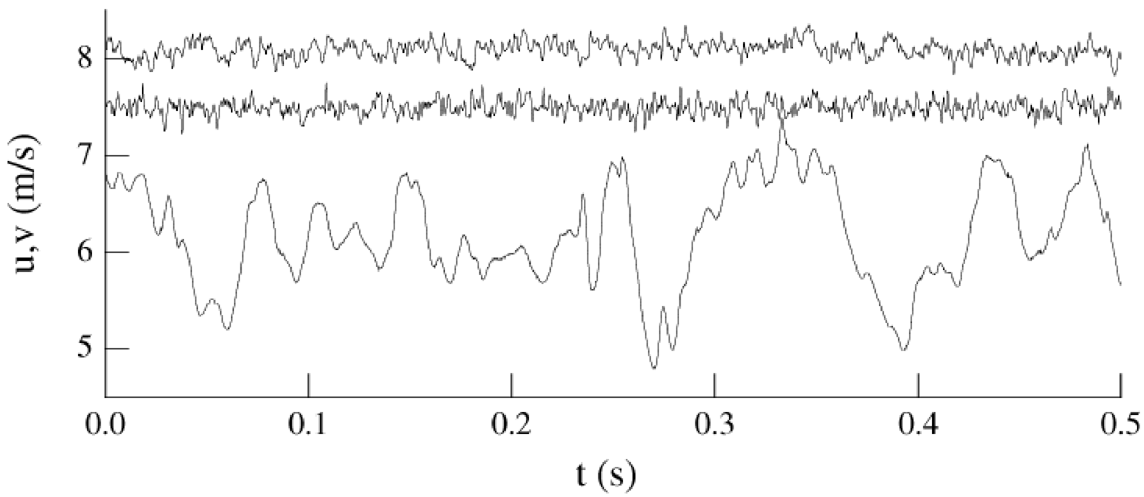
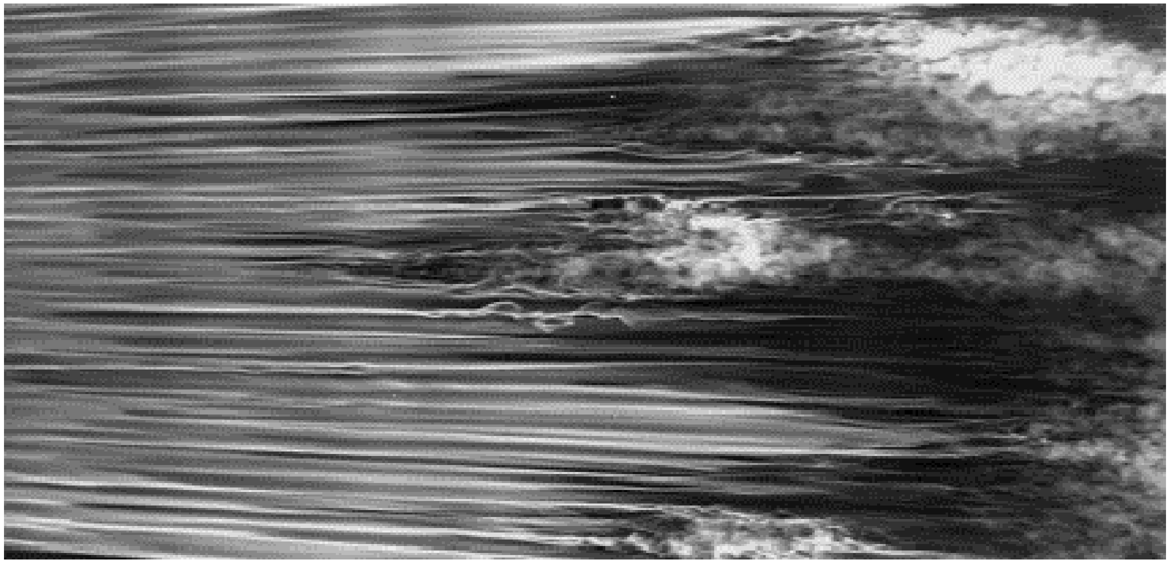


Reynolds pipe flow experiment



FREE-STREAM TURBULENCE AND STREAK BREAKDOWN





WEAKLY NONLINEAR ANALYSIS

$$\frac{Du_r}{Dt} = \frac{u_\theta^2}{r} - \frac{1}{\rho}\frac{\partial P}{\partial r} + \nu\left(\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r}\frac{\partial u_\theta}{\partial \theta}\right)$$

$$\frac{Du_\theta}{Dt} = -\frac{1}{\rho r}\frac{\partial P}{\partial \theta} + \nu\left(\nabla^2 u_\theta - \frac{u_\theta}{r^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right)$$

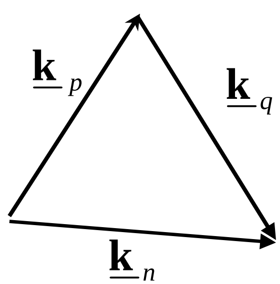
$$\frac{Du_z}{Dt} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \nu\nabla^2 u_z$$

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

$$\mathbf{u}(\underline{\mathbf{x}}, t) = \sum_n \mathbf{a}_n(t)\cos\left(\underline{\mathbf{k}}_n \cdot \underline{\mathbf{x}} + \omega_n t\right)$$

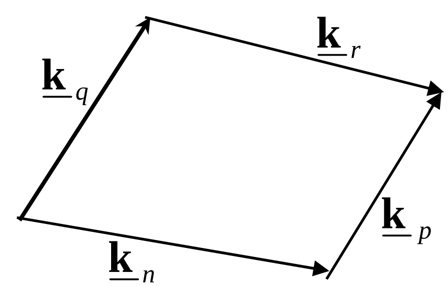
$$\frac{da_n}{dt} + i\omega_n a_n = \varepsilon \sum_{p,q,r} Q_{npq} a_p^* a_q + \varepsilon^2 \sum_{p,q,r} T_{npqr} a_p^* a_q a_r$$

Triad interaction



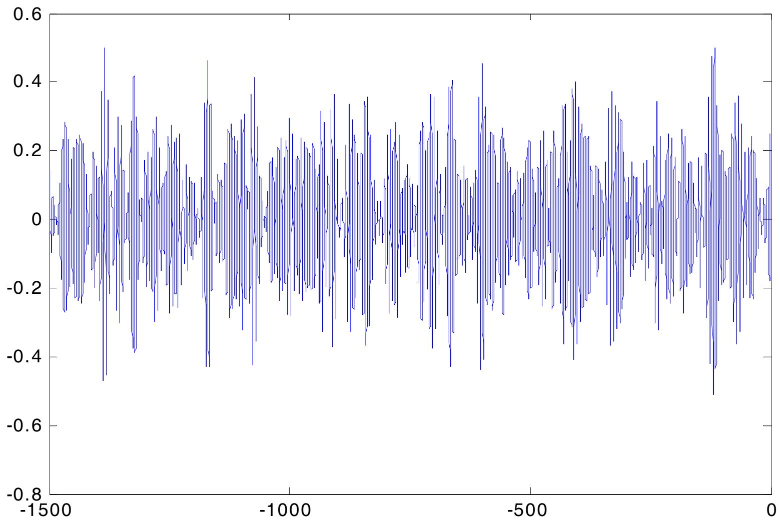
$$\underline{\mathbf{k}}_n = \underline{\mathbf{k}}_q + \underline{\mathbf{k}}_r$$

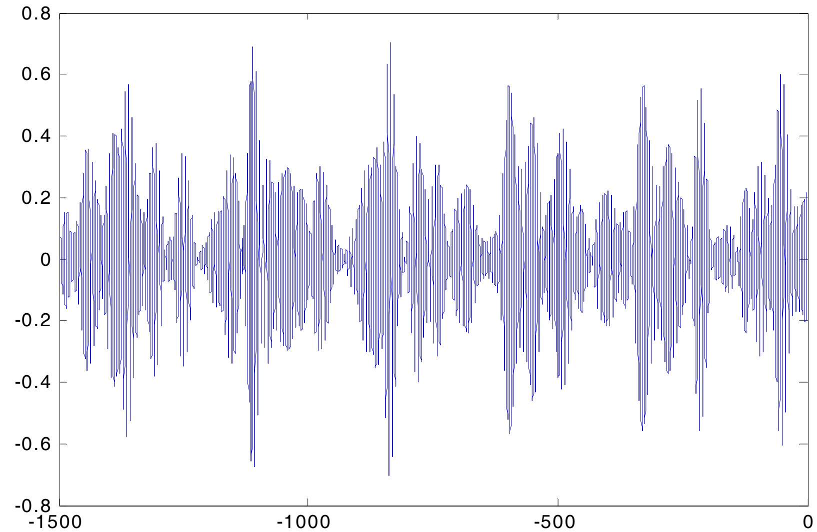
Quartet interaction

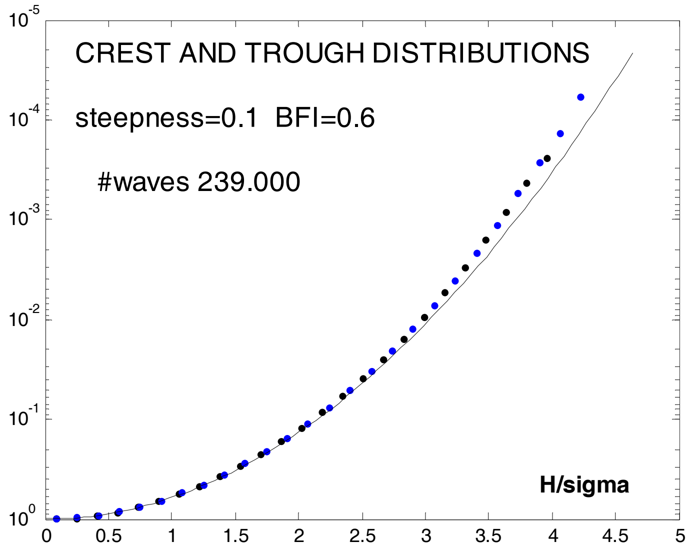


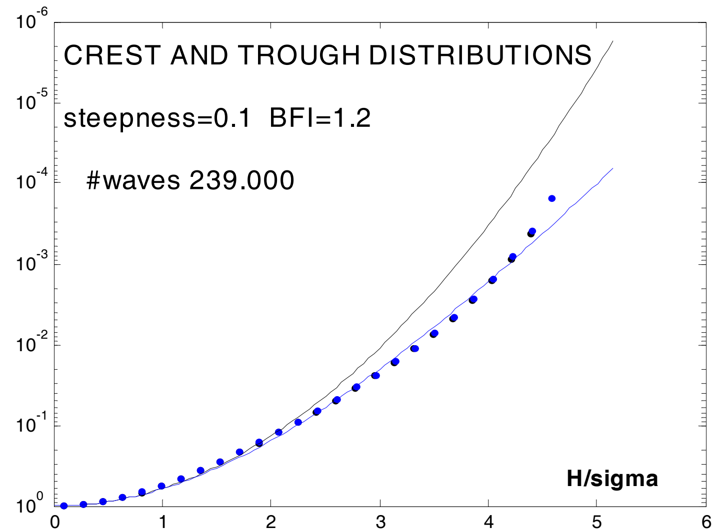
$$\underline{\mathbf{k}}_n + \underline{\mathbf{k}}_p = \underline{\mathbf{k}}_q + \underline{\mathbf{k}}_r$$

MONTECARLO SIMULATIONS FOR QUARTET INTERACTIONS

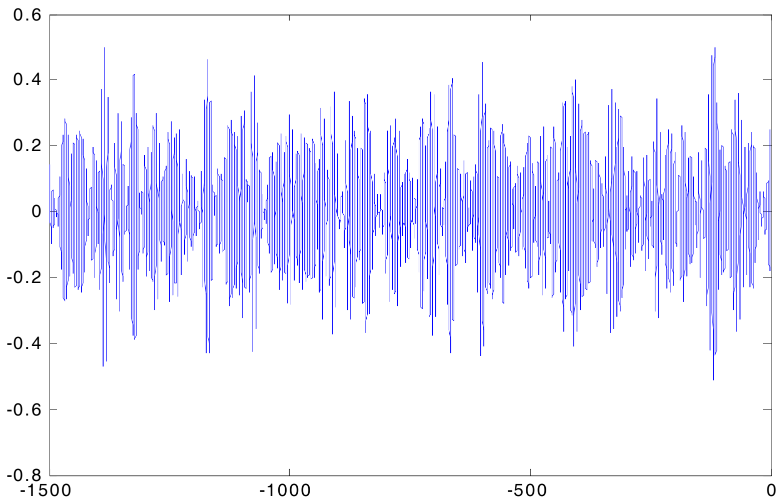


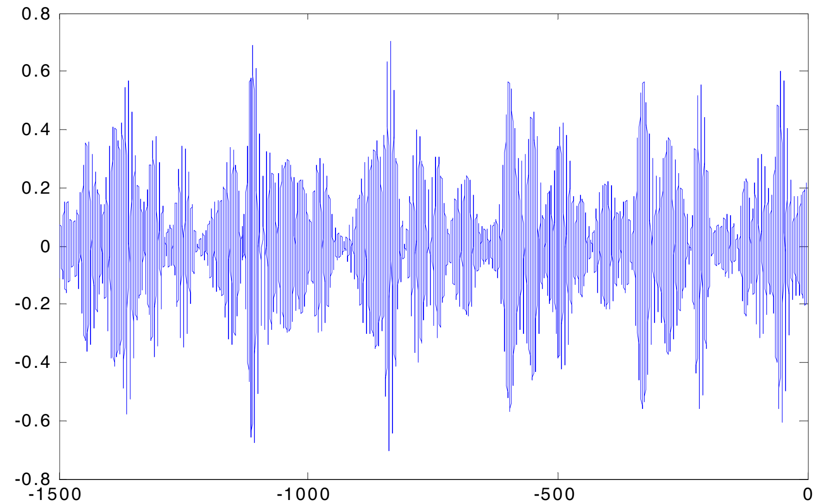


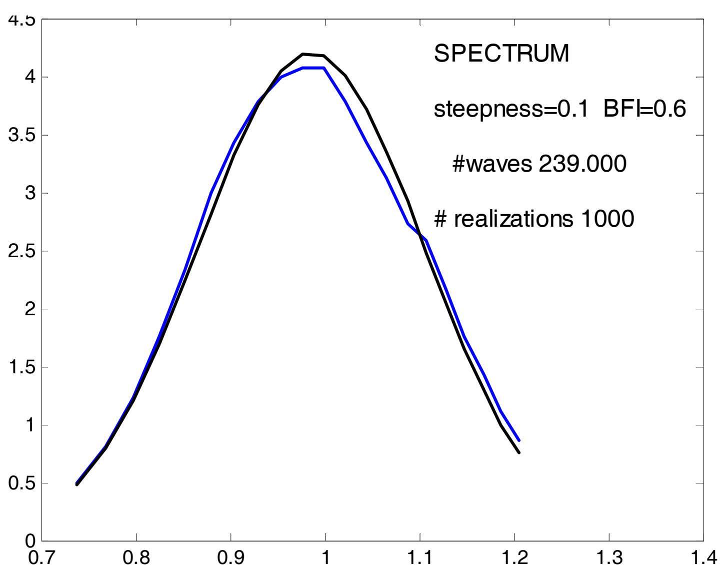


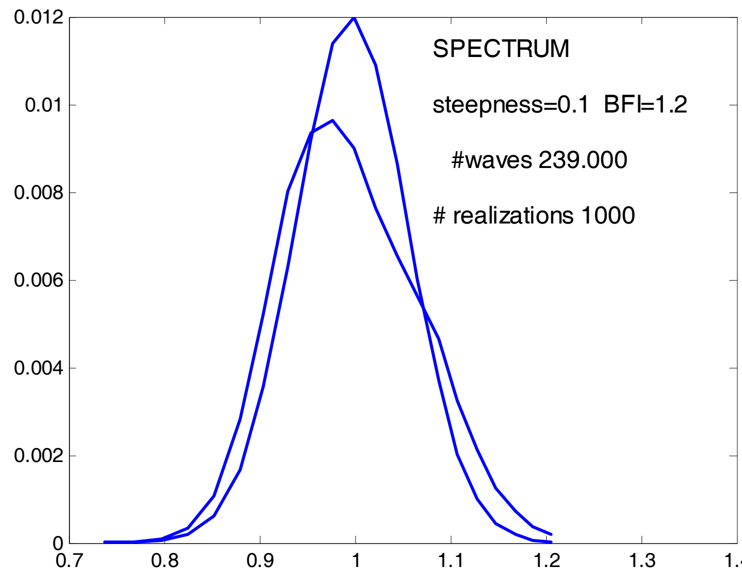


ENERGY SPECTRUM

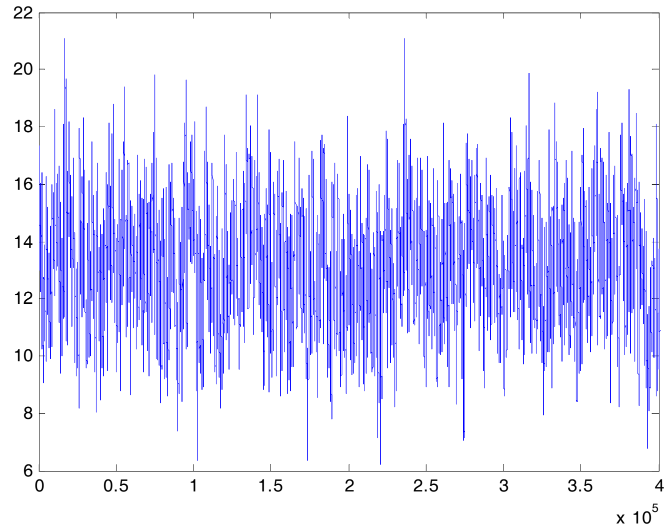


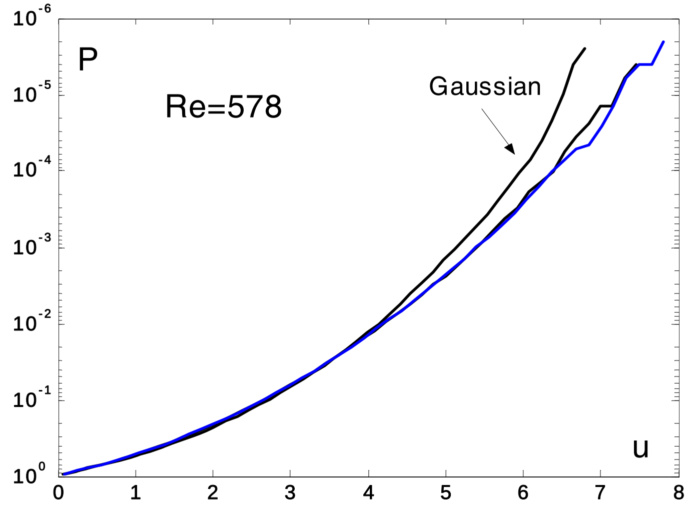






Experimental data





Successive Wave Crests in Gaussian Seas

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