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NON-LINEAR SPACE-TIME EVOLUTION OF A HIGH WAVE CREST

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ABSTRACT

The quasi-determinism theory, for the mechanics of the linear three-dimensional waves, was obtained by Boccotti in the eighties. The first formulation of the theory deals with the largest crest amplitude; the second formulation deals with the largest wave height. The theory was verified in the nineties with some small scale field experiments.

In this paper the first formulation of the quasi-determinism theory, for the space-time domain, is extended to the second order. The analytical expressions of the second-order free surface displacement and velocity potential are obtained. Therefore the space-time evolution of a wave group, to the second-order in a Stokes expansion, when a very large crest occurs at a fixed time and location is investigated.

Finally the second-order probability of exceedance of the crest amplitude is obtained, as a function of two deterministic parameters.

KEYWORDS

Surface displacement, velocity potential, second-order, wave crest, wave group, probability of exceedance.

INTRODUCTION

The quasi-determinism theory for the mechanics of linear random waves was formulated by Boccotti in the eighties. The first formulation of the theory (Boccotti [1-2]) deals with the highest crest; the second formulation (Boccotti [3-5]) deals with the highest crest-to-trough wave height. The theory was verified in the nineties, either for waves in an undisturbed field or for waves interacting with structures, with some small-scale field experiments (Boccotti [6-7], Boccotti et al., [8-9]). Boccotti [10] proposed then a complete review of the theory, and showed as the two formulations are congruent to each other.

An alternative approach for the derivation of the quasi-determinism theory was proposed by Phillips et al. ([11-12]), which obtained also a field verification off the Atlantic coast of USA.

The first formulation of the theory (derived only for the time domain) was also given by Tromans et al. ([13]), who renamed the theory as 'New wave'.

Following the first formulation of the quasi-determinism theory we have that, if a very high crest (very high with respect to the mean crest height) occurs in some time and location in a Gaussian sea state, this implies that a well-defined quasi-deterministic wave group generates the highest crest.

In this paper the quasi-determinism theory is extended to the second-order in a Stokes expansion: the second-order free surface displacements $\overline{\eta}$ and velocity potential $\overline{\phi}$, when a very high crest occurs at a fixed time and location, are obtained. In particular, if the high wave crest occurs at point x_o at instant t_o, the analytical second-order expressions of $\overline{\eta}$ and $\overline{\phi}$ (at any depth z) are derived at any point $x_o + X$ at any instant $t_o + T$, as a function of the wave spectrum.

The linear and nonlinear predictions are then compared, showing as the second order effects modify the wave profile.

Finally, the second-order probability of exceedance of the largest crest height is obtained. For the case of deep water the later probability depends upon a characteristic wave steepness and the wave spectrum.

THE QUASI-DETERMINISM THEORY FOR A LINEAR HIGH CREST

The quasi-determinism theory by Boccotti ([10]), for linear three dimensional wave groups (either for high wave height or for high wave crest) may be used in place of the periodic waves: it enables us to predict the space time evolution of the free surface displacement and of the velocity potential when a very high wave occurs. The theory may be applied either for waves in an undisturbed field or for waves interacting with structures, in both the formulations. The first formulation of the quasi-determinism theory deals with the crest height, the second formulation with the crest-to-through height.

In this paper we shall apply only the first formulation particularized for long-crested random waves. In detail we have that, if a local wave maximum of given elevation H_C occurs at a time t_o at a fixed point x_o, and if $H_C/\sigma \to \infty$ (σ being the standard deviation of the free surface displacement), with probability approaching 1 the surface displacement at point $x_o + X$ at time $t_o + T$ is asymptotically equal to the deterministic form

$$\bar{\eta}_I(x_o + X, t_o + T) = \frac{\Psi(X,T;x_o)}{\Psi(0,0;x_o)} H_C \quad \text{if } H_C/\sigma \to \infty . \quad (1)$$

Let us note that the theory is exact for $H_C/\sigma \to \infty$, that is for the crest height very large with respect to the mean crest height. The space-time covariance $\Psi(X,T;x_o)$ is defined as

$$\Psi(X,T;x_o) \equiv < \eta(x_o,t)\eta(x_o + X, t+T) >, \quad (2)$$

where the ensemble average operator $< \cdot >$ is defined as

$$< f(t) > = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau f(t)\mathrm{d}t . \quad (3)$$

Because the absolute maximum of the autocovariance function $\psi(T)$ is at $T = 0$, the local wave maximum of given elevation H_C is the highest maximum of its wave. We have also a symmetric $\bar{\eta}_I$ profile both in time domain (for X=0) and in the space domain (for T=0).

The velocity potential of the deterministic wave group (1), at point $x_o + X$, at depth z, is given by:

$$\bar{\phi}_I(x_o + X, z, t_o + T) = \frac{\Phi(X,z,T;x_o)}{\Psi(0,0;x_o)} H_C \quad (4)$$

where the space-time covariance $\Phi(X,z,T;x_o)$ is defined as

$$\Phi(X,z,T;x_o) \equiv < \eta(x_o,t)\phi(x_o + X, z, t+T) > . \quad (5)$$

The linear deterministic wave group in an undisturbed field

For an undisturbed wave field the free surface displacement [see Eq. (1)] of the wave group at $x_o + X$ at time $t_o + T$, when an exceptional crest height of given elevation H_C occurs at time t_o at fixed point x_o, may be rewritten as a function of the frequency spectrum $E(\omega)$:

$$\bar{\eta}_I(x_o + X, t_o + T) = \frac{H_C}{\sigma^2} \int_0^\infty E(\omega)\cos(kX - \omega T)\, \mathrm{d}\omega \quad (6)$$

where

$$\sigma^2 = \int_0^\infty E(\omega)\, \mathrm{d}\omega . \quad (7)$$

As function of the frequency spectrum, the velocity potential at a fixed point (x_o +X, z), when a very large crest occurs at point x_o, is given by:

$$\bar{\phi}_I(x_o + X, z, t_o + T) = = g \frac{H_C}{\sigma^2} \int_0^\infty E(\omega)\omega^{-1} \exp(kz)\sin(kX - \omega T)\mathrm{d}\omega \quad (8)$$

where the wave number k, in deep water, is equal to ω^2/g. From Eq. (8) we may obtain the linear wave kinematics of wave group as well as the first Stokes order pressure fluctuation $\Delta\bar{p}_I$ (which is defined as $\Delta\bar{p}_I = -\rho\partial\bar{\phi}_I/\partial t$).

NON-LINEAR SPACE-TIME EVOLUTION OF A HIGH WAVE CREST

According to the theory of wind generated waves, the free surface displacement to the first order in a Stokes expansion is a random Gaussian process. It may be modelled as a sum of a very large number of periodic components, with phase angles randomly and uniformly distributed between 0 and 2π. The wave amplitudes follow the Rayleigh distribution.

To the second order in a Stokes expansion, the free surface displacement and the velocity potential, for long-crested random deep-water waves, are respectively given by (Sharma & Dean [14], Tayfun [15]):

$$\eta(x,t) = \eta_1 + \eta_2 = \sum_{n=1}^N a_n \cos\psi_n + \frac{1}{4}\sum_{n=1}^N \sum_{m=1}^N a_n a_m \cdot \cdot \left[(k_n + k_m)\cos(\psi_n + \psi_m) - |k_n - k_m|\cos(\psi_n - \psi_m)\right] \quad (9)$$

$$\phi(x,z,t)=\phi_1+\phi_2=g\sum_{n=1}^{N}a_n\omega_n^{-1}\exp(k_n z)\sin\psi_n + -\sum_{n=1}^{N}\sum_{m=n}^{N}a_n a_m\omega_m\exp[(k_m-k_n)z]\sin(\psi_m-\psi_n) \tag{10}$$

where

$$\psi_n=k_n x-\omega_n t+\varepsilon_n \tag{11}$$

and $\{a_n\}_{n\in\aleph}$, $\{\varepsilon_n\}_{n\in\aleph}$ coefficients to be specified.

In the following we shall define the conditions under which $\eta(x,t)$, defined above by Eq. (9), has a local maximum at point $x=x_o$ at time instant $t=t_o$. By means of the quasi-determinism theory this maximum is the crest of its own wave. Thus the deterministic wave group solution to the second order is derived.

The deterministic wave group to the second order in a Stokes expansion: the free surface displacement

Let us assume that the free surface displacement has a local maximum h at point $x=x_o$ and that this maximum occurs at time $t=t_o$. We have to obtain the deterministic free surface displacement $\overline{\eta}(X,T)$ at point x_o+X at time instant t_o+T, when h is very large with respect to the standard deviation σ of the free surface displacement.

The conditions of a local maximum at time $t=t_o$ (that is T=0) at point $x=x_o$ (that is $X=0$) are

$$\overline{\eta}(X,T)\ \text{such that}\ \begin{cases}\overline{\eta}\,|_{X=0,T=0}=h\\ (\partial\overline{\eta}/\partial x)\big|_{X=0,T=0}=0\\ (\partial^2\overline{\eta}/\partial x^2)\big|_{X=0,T=0}<0\end{cases} \tag{12}$$

By applying a perturbation approach we expand the assigned height h as

$$h=h_0+h_1+h_2+.......... \tag{13}$$

where h_0, h_1, h_2, … are unknown parameters to be determined. We assume that $h_0\propto\sigma$, $h_1\propto\sigma^2$, …, $h_n\propto\sigma^{n+1}$,..., where σ is the standard deviation of the surface displacement. From the general solution (9), conditions (12) give three equations, which are respectively

$$\sum_{n=1}^{N}a_n\cos\vartheta_n+\frac{1}{4}\sum_{n=1}^{N}\sum_{m=1}^{N}a_n a_m\big[(k_n+k_m)\cos(\vartheta_n+\vartheta_m)+ -|k_n-k_m|\cos(\vartheta_n-\vartheta_m)\big]=h_0+h_1+h_2+......... \tag{14}$$

$$-\sum_{n=1}^{N}a_n\,k_n\sin\vartheta_n+\frac{1}{4}\sum_{n=1}^{N}\sum_{m=1}^{N}a_n a_m\big[-(k_n+k_m)^2\cdot \cdot\sin(\vartheta_n+\vartheta_m)++|k_n-k_m|(k_n-k_m)\sin(\vartheta_n-\vartheta_m)\big]=0 \tag{15}$$

$$-\sum_{n=1}^{N}a_n\,k_n^2\cos\vartheta_n+\frac{1}{4}\sum_{n=1}^{N}\sum_{m=1}^{N}a_n a_m\big[-(k_n+k_m)^3\cdot \cdot\cos(\vartheta_n+\vartheta_m)+|k_n-k_m|(k_n-k_m)^2\cos(\vartheta_n-\vartheta_m)\big]<0 \tag{16}$$

where $\vartheta_n=k_n x_0-\omega_n t_0+\varepsilon_n$. Because we assume $a_n\propto\sigma$, a hierarchy of perturbation equations to the first and to the second order in a Stokes expansion can be obtained. All the terms in the h expansion higher than the second order vanish.

i) Perturbation equations to $O(\sigma)$

To the first order, Equations (14-16) give respectively

$$O(\sigma)\quad\begin{cases}\sum_{n=1}^{N}a_n\cos\vartheta_n=h_0\\ \sum_{n=1}^{N}a_n\,k_n\sin\vartheta_n=0\\ \sum_{n=1}^{N}a_n\,k_n^2\cos\vartheta_n>0\end{cases} \tag{17}$$

The second and third Equations in (17) are satisfied if $\vartheta_n=0\ \ \forall n$ whatever are the values of the coefficients $\{a_n\}_{n\in\aleph}$ which gives a constraint from the first equation as $h_0=\sum_n a_n\cos\vartheta_n$. Really other solutions could exist with some coefficients $\vartheta_n\neq 0$.

From the quasi-determinism theory we know that if a very large crest height $H_C=h_0$ occurs at a fixed point $x=x_o$ at time instant $t=t_o$, the free surface displacement [Eq. (6)] in discrete form is given by

$$\overline{\eta}_I(X,T)=\sum_{n=1}^{N}\tilde{a}_n\cos\tilde{\psi}_n \tag{18}$$

where

$$\tilde{a}_n = \frac{h_0}{\sigma^2} E(\omega_n)\mathrm{d}\omega_n \qquad (19)$$

and

$$\tilde{\psi}_n = k_n X - \omega_n T \ . \qquad (20)$$

Because the high wave group defined by Eq. (18) at $(X = 0, T = 0)$ attains a maximum, it follows:

$$(\partial \overline{\eta}_I / \partial X)_{X=0,T=0} = -\sum_{n=1}^{N} \tilde{a}_n k_n \sin\tilde{\psi}_n = 0 \qquad (21)$$

$$(\partial^2 \overline{\eta}_I / \partial X^2)_{X=0,T=0} = -\sum_{n=1}^{N} \tilde{a}_n k_n^2 \cos\tilde{\psi}_n < 0 \qquad (22)$$

(because at X=0, T=0 we have $\tilde{\psi}_n = 0 \ \ \forall n$) and

$$\overline{\eta}_I (X = 0, T = 0) = \sum_{n=1}^{N} \tilde{a}_n = h_0 \ . \qquad (23)$$

Finally, by comparing these equations (21-23) with equations of system (17), we have that the solution of the later, for $h_0/\sigma \to \infty$, is $\psi_n = \tilde{\psi}_n$, $a_n = \tilde{a}_n$ [being $\tilde{a}_n$ given by Eq. (19)], which implies $\vartheta_n = 0 \ \ \forall n$.

ii) The second order problem

To the second order Equations (14-16) give

$$O(\sigma^2) \begin{cases} \frac{1}{4}\sum_{n,m} a_n a_m \left[(k_n + k_m)\cos(\vartheta_n + \vartheta_m) + \right. \\ \left. \qquad - |k_n - k_m| \cos(\vartheta_n - \vartheta_m) \right] = h_1 \\ \frac{1}{4}\sum_{n,m} a_n a_m \left[-(k_n + k_m)^2 \sin(\vartheta_n + \vartheta_m) + \right. \\ \left. \qquad + |k_n - k_m| (k_n - k_m) \sin(\vartheta_n - \vartheta_m) \right] = 0 \\ \frac{1}{4}\sum_{n,m} a_n a_m \left[-(k_n + k_m)^3 \cos(\vartheta_n + \vartheta_m) + |k_n - k_m| \cdot \right. \\ \left. \cdot (k_n - k_m)^2 \cos(\vartheta_n - \vartheta_m) \right] < \sum_{n=1}^{N} a_n k_n^2 \cos\vartheta_n \end{cases} \qquad (24)$$

From the first order problem (17), it has been shown that $\vartheta_i = 0 \ \ \forall i$, which implies that the last two conditions in Eq. (24) are satisfied, while the first condition becomes of the form

$$h_1 = \frac{1}{4}\sum_{n,m} a_n a_m \left[(k_n + k_m) - |k_n - k_m| \right]. \qquad (25)$$

By considering Eq. (19), which defines a_n, we obtain, in continuous form

$$h_1 = \frac{H_C^2}{4\sigma^4} \int_0^\infty \int_0^\infty E(\omega_1)E(\omega_2) \left[(k_1 + k_2) - |k_1 - k_2| \right] \mathrm{d}\omega_1 \mathrm{d}\omega_2 \ . \qquad (26)$$

Finally, we have that, if a very large crest height occurs, the second order height may be written as:

$$h = H_C + \frac{H_C^2}{4\sigma^4} \int_0^\infty \int_0^\infty E(\omega_1)E(\omega_2) \left[(k_1 + k_2) + \right. \\ \left. - |k_1 - k_2| \right] \mathrm{d}\omega_1 \mathrm{d}\omega_2 + o(\sigma^2) \ . \qquad (27)$$

More in general, the second order free surface displacement, when a very high crest occurs at time instant t_o at point x_o, is given by:

$$\overline{\eta}(X,T) = \overline{\eta}_I + \overline{\eta}_{II} = \frac{H_C}{\sigma^2} \int_0^\infty E(\omega) \cos\psi \, \mathrm{d}\omega + \\ + \frac{H_C^2}{4\sigma^4} \int_0^\infty \int_0^\infty E(\omega_1)E(\omega_2) \left[(k_1 + k_2)\cos(\psi_1 + \psi_2) + \right. \\ \left. - |k_1 - k_2| \cos(\psi_1 - \psi_2) \right] \mathrm{d}\omega_1 \mathrm{d}\omega_2 \qquad (28)$$

[let us note that σ^2 is defined by Eq. (7)].

The deterministic wave group to the second order in a Stokes expansion: the velocity potential

Regarding the velocity potential of the linear wave group (8), it can be written in discrete form as

$$\overline{\phi}_I (X, z, T) = g \sum_{n=1}^{N} a_n \, \omega_n^{-1} \exp(k_n z) \sin\psi_n \ . \qquad (29)$$

By considering the general second order solution for the velocity potential, given by Eq. (10), we obtain the second order velocity potential of the wave group in continuous form as

$$\bar{\phi}(X,z,T)=\bar{\phi}_I+\bar{\phi}_{II}=$$

$$=\frac{g\,H_C}{\sigma^2}\int_0^\infty E(\omega)\omega^{-1}\exp(kz)\sin\psi\,\mathrm{d}\omega+\frac{H_C^2}{\sigma^4}\cdot \qquad (30)$$

$$\cdot\int_0^\infty\int_{\omega_1}^\infty E(\omega_1)E(\omega_2)\,\omega_2\exp[(k_2-k_1)z]\sin(\psi_1-\psi_2)\mathrm{d}\omega_2\mathrm{d}\omega_1\,.$$

CALCULATION OF THE SECOND ORDER $\bar{\eta}$ AND $\bar{\phi}$

The expressions (28) and (30) may be rewritten as a function of the nondimensional frequency spectrum. In particular, for a JONSWAP spectrum (Hasselmann et al. [16]) we have

$$E(w\omega_p)=\alpha g^2\omega_p^{-5}E_a(w)\,, \qquad (31)$$

where α is the Phillips parameter, $\omega_p=2\pi/T_p$ the dominant frequency and

$$E_a(w)=w^{-5}\exp\left[-1.25w^{-4}\right]\exp\left\{\ln\chi_1\exp\left[-\frac{(w-1)^2}{2\chi_2^2}\right]\right\} \qquad (32)$$

is the non dimensional spectrum (being $w_j=\omega_j/\omega_p$). For a mean JONSWAP spectrum, the parameters χ_1 and χ_2 are equal respectively to 3.3 and 0.08.

Furthermore, by defining $k_j=k_{w\,j}2\pi/L_{p_o}$, where L_{p_o} is the dominant wavelength in deep water [$L_{p0}\equiv gT_p^2/(2\pi)$], the non dimensional wave number is $k_{w\,j}=w_j^2$ and therefore $\psi_j=k_{w\,j}2\pi X/L_{p_o}-2\pi w_jT/T_p$.

It follows that

$$\bar{\eta}(X,T)=\frac{H_C}{\sigma_w^2}\int_0^\infty E_a(w)\cos\psi\,\mathrm{d}w+$$

$$+\frac{H_C^2}{4\sigma_w^4}\frac{\omega_p^2}{g}\int_0^\infty\int_0^\infty E_a(w_1)E_a(w_2)\left[\left(w_1^2+w_2^2\right)\cos(\psi_1+\psi_2)+\right. \qquad (33)$$

$$\left.-\left|w_1^2-w_2^2\right|\cos(\psi_1-\psi_2)\right]\mathrm{d}w_1\mathrm{d}w_2$$

$$\bar{\phi}(X,z,T)=\frac{g\,H_C}{\sigma_w^2}\omega_p^{-1}\int_0^\infty E_a(w)w^{-1}\exp(2\pi\,k_wz/L_{p0})\cdot$$

$$\cdot\sin\psi\,\mathrm{d}w+\frac{H_C^2}{\sigma_w^4}\omega_p\int_0^\infty\int_{\omega_1}^\infty E_a(w_1)E_a(w_2)\,w_2\cdot \qquad (34)$$

$$\cdot\exp[(k_{w2}-k_{w1})2\pi\,z/L_{p0}]\sin(\psi_1-\psi_2)\mathrm{d}w_2\mathrm{d}w_1$$

where

$$\sigma_w^2=\int_0^\infty E_a(w)\mathrm{d}w=\sigma^2/(\alpha g^2\omega_p^{-4})\,. \qquad (35)$$

Let us note that, from second order velocity potential (34), we may easily derive the wave pressure and the wave kinematics, exact to the second order in a Stokes expansion (see Applications). For example, the second order pressure fluctuation is $\Delta\bar{p}=\Delta\bar{p}_I+\Delta\bar{p}_{II}$, where the second order component $\Delta\bar{p}_{II}$ is easily derived by the formula

$$\Delta\bar{p}_{II}=-\rho\frac{\partial\bar{\phi}_{II}}{\partial t}-\frac{1}{2}\rho\left[\left(\frac{\partial\bar{\phi}_I}{\partial x}\right)^2+\left(\frac{\partial\bar{\phi}_I}{\partial z}\right)^2\right] \qquad (36)$$

(see Applications).

APPLICATIONS

The wave crest evolution in space domain

Figure 1 shows the space-time evolution of the second order free surface displacement, when a very large crest height occurs at point x_o at time instant t_o. In particular it is shown the deep water free surface displacement *in the space domain* at different time instant, to the second-order predictions, obtained by means of Eq. (33).

We find a well defined wave group, which moves along the *x*-axis, crossing the point x_o (where X=0). We have also that propagation speed for individual waves is greater than the wave group celerity: each wave 'runs along the envelope from the tail where it is born to the head where it dies' (Boccotti [10]).

The wave group shows also firstly a development stage, during which the height of the largest crest (at that fixed instant) increases; therefore at time t_o we have the apex of the group development: at this time the wave crest at point x_o reaches its maximum. All these results on the space time evolution of non-linear wave groups are identical to those of Boccotti's book ([10]), from the first order quasi-determinism theory.

As for the second order effects, they increase the crest amplitude and decrease the trough amplitude. For example we

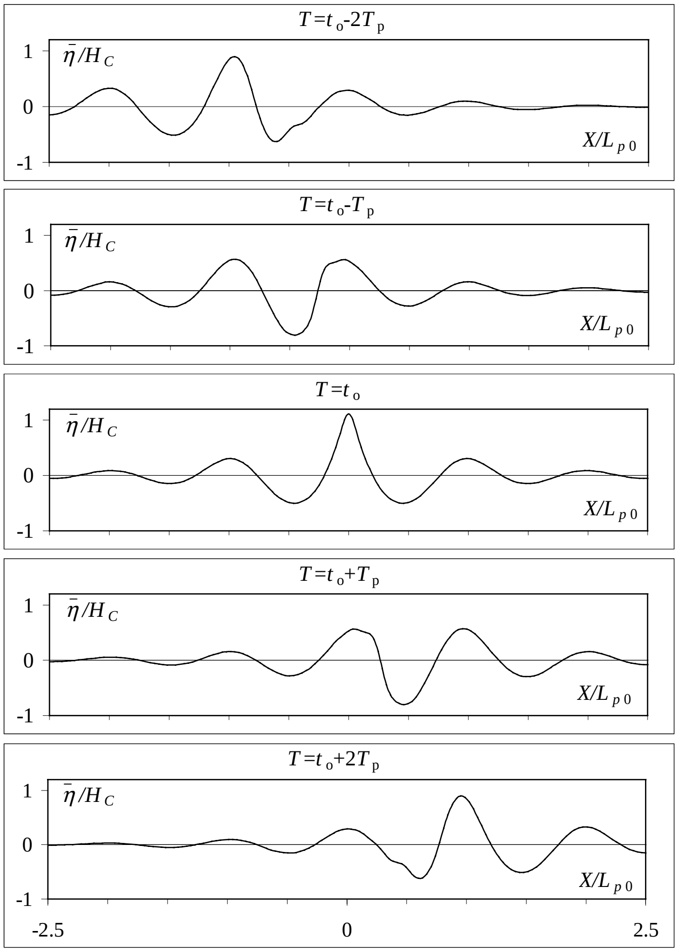

Figure 1. The second-order space-time evolution of a wave group in which a very large crest occurs at (X=0, T=t_o).

have that, *in space domain*, at time t_o, second-order largest crest and trough amplitudes are equal respectively to 1.11 and 0.94 times the linear predictions; the ratio between crest and trough amplitude is equal to 1.87 to the first order and to 2.22 to the second-order.

The second-order wave crest evolution in time domain

Figures 2-5 show a wave group *in time domain*. Figure 2 shows the wave group at point x_o, where the largest crest occurs. Dotted line gives the linear prediction, which is obtained from Eq. (6): we have the well known symmetric profile ('New wave'), obtained also by Tromans et al. ([13]). In this case the largest trough amplitude is equal to ψ^* times the largest crest amplitude, being ψ^* the narrow bandedness parameter defined by Boccotti ([3-4]). The parameter ψ^* is defined as the absolute value of the quotient between the absolute minimum and the absolute maximum of the autocovariance function. It is equal to 0.73 for the mean JONSWAP spectrum and to 0.67 for a Pierson-Moskowitz frequency spectrum. The quotient between crest and trough amplitudes is then 1.37 and 1.49 respectively.
To the second order, *in time domain*, the quotient between crest and trough amplitudes is equal to 1.67 for a mean JONSWAP spectrum and to 1.85 for a Pierson-Moskowitz spectrum. The second order wave profile $\overline{\eta}(x_o,t)$ (being $\overline{\eta} = \overline{\eta}_1 + \overline{\eta}_2$) is furthermore symmetric, because the component $\overline{\eta}_2$ is symmetric and presents the highest wave crest in phase with the linear highest crest of $\overline{\eta}_1$.

Figure 3 shows the second order wave pressure $\Delta\overline{p}$ at point $(x_o, z = -0.05L_{p0})$, when a very large crest of free surface displacement occurs at point x_o. As we can see second-order effects reduce the amplitude of the highest crest of linear component $\Delta\overline{p}_I$.

How explain some asymmetries in the time domain wave profile

By applying the first formulation of the quasi-determinism theory we may obtain the free surface displacement in time domain at any fixed point $x_o + X$, if a very large crest occurs at point x_o. In Fig. 2 we have seen the linear wave profile at point x_o in time domain, which is a symmetric profile ('New wave').
In this paper we have extended the quasi-determinism theory to the second order in a Stokes expansion. We have obtained that the second-order wave profile is symmetric too, either in space domain for T=0 or in time domain for X=0. Now we analyze the wave profile $\overline{\eta}\ (T)$ at some points close to x_o. Figure 4 shows the $\overline{\eta}\ (T)$ at points $X/L_{p0} = (-0.10, -0.05, 0)$. As we can see, for $X < 0$ the profile is not symmetric: the trough depths before (H_{T1}) and after (H_{T2}) the highest crest are different, with $H_{T1} > H_{T2}$ (let us note that at points having $X/L_{p0} > 0$ we have $H_{T1} < H_{T2}$, as we can see from Figure 5). Therefore, an asymmetry in wave profile may be explained by analyzing the space time evolution of the wave groups: if we have a time record with a very high crest, with $H_{T1} > H_{T2}$ ($H_{T1} < H_{T2}$), we have probably recorded the highest crest at a point X<0 (X>0), before (after) point x_o where the wave groups reaches the apex of its development.
Finally, it is easy to verify that in deep water these asymmetries are only slightly a non-linear effects: they may be obtained from the linear quasi-determinism theory, from Eq. (6).

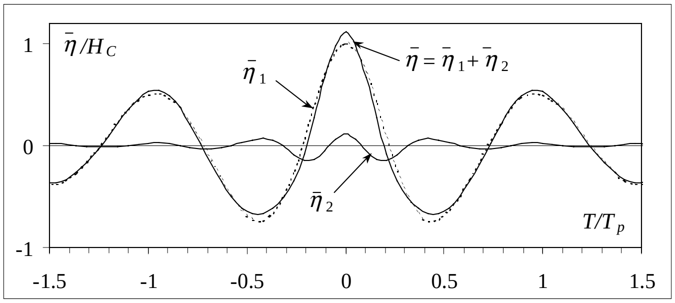

Figure 2. The second-order time evolution of a wave group in which a very large crest occurs. Continuous lines give the second-order prediction [Eq. (33)]. Dotted line gives linear prediction [Eq. (6)], which is the 'New wave' symmetric wave profile.

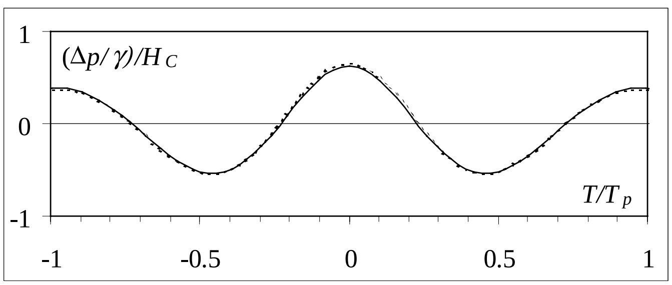

Figure 3. The time domain second-order wave pressure Δp at point $(x_o, z/L_{p0} = -0.05)$, when a very large crest height H_C of free surface displacement occurs at $(x_o, T = 0)$ (see Figure 2). Continuous lines give the second-order prediction. Dotted line gives linear prediction.

THE ASYMPTOTIC FORM OF THE SECOND-ORDER PROBABILITY OF EXCEEDANCE OF THE CREST HEIGHT

The second-order crest elevation, given by Equations (13) and (26), may be rewritten as

$$h = H_C + \varphi \frac{H_C^2}{\sigma} \qquad (36)$$

where

$$\varphi = \frac{\varepsilon_p}{4\sigma_w^4} \int_0^\infty \int_0^\infty E_a(w_1) E_a(w_2) \left[\left(w_1^2 + w_2^2 \right) - \left| w_1^2 - w_2^2 \right| \right] \mathrm{d}w_1 \mathrm{d}w_2 \qquad (37)$$

being $\varepsilon_p = k_p \sigma$ the wave steepness.

The variance of the second order process is easily derived from (9) and has expression as

$$\sigma_\eta^2 = \frac{\sigma^2}{\beta^2} \qquad (38)$$

where:

$$\beta = \left[1 + \frac{\varepsilon_p^2}{2\sigma_w^4} \int_0^\infty \int_0^\infty E_a(w_1) E_a(w_2) \left(w_1^4 + w_2^4 \right) \mathrm{d}w_1 \mathrm{d}w_2 \right]^{-1/2} \qquad (39)$$

So we have that the non-dimensional crest height $\xi_{high} = h/\sigma_\eta$ can be expressed as the following

$$\xi_{high} = \beta u + \varphi \beta u^2 \qquad (40)$$

where the random variable $u = H_C/\sigma$ has Rayleigh distribution. As consequence the probability of exceedance of the absolute maximum (crest) is:

$$P\left(\xi_{high} > \xi\right) = \exp\left[-\frac{1}{8\varphi^2} \left(1 - \sqrt{1 + \frac{4|\varphi|\xi}{\beta}} \right)^2 \right]. \qquad (41)$$

The probability $P(\xi_{high} > \xi)$, which is valid for $\xi \to \infty$, depends then upon the two parameters φ and β.

The distribution of the crest of the highest waves: analytical prediction and comparison with data

The analytical prediction of the crest height distribution are then compared with the data of numerical simulations. In detail a second-order simulation of random waves with a mean JONSWAP spectrum has been carried out, with a generation of near 50000 waves.

Fig. 6 shows the crest height distribution, obtained from data, and theoretical predictions, which are obtained from Eq. (41). Let us note that for a mean JONSWAP spectrum, in deep water we have $\varphi = 0.028$ and $\beta = 0.9996$.

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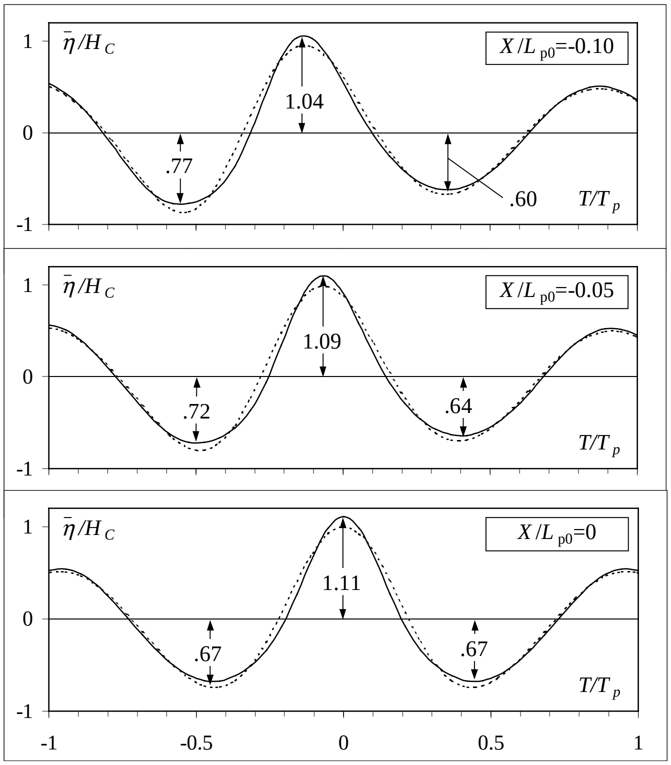

Figure 4. The second-order time evolution of a wave group at fixed points X/L_{p_0}, when a very large crest occurs at (X=0,T=0). Dotted lines show linear predictions, obtained from Boccotti's quasi-determinism theory (first formulation).

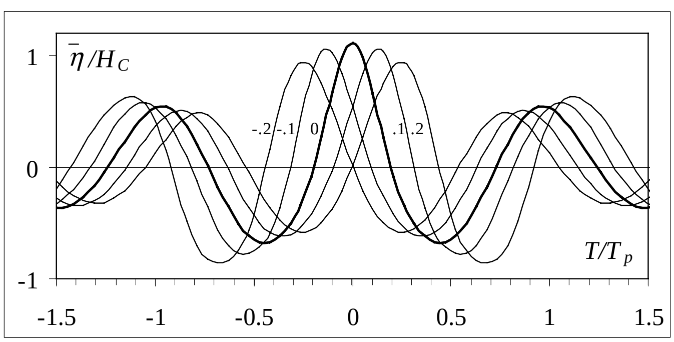

Figure 5. The second-order time evolution of a wave group at fixed points X/L_{p_0} (-0.20, -0.10, 0, 0.10, 0.20) when a very large crest occurs at (X=0,T=0).

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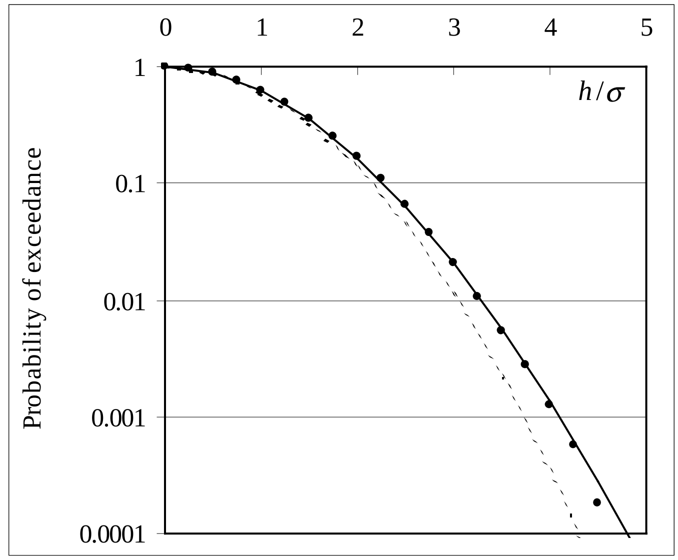

Figure 6. The second-order probability of exceedance of the crest height (continuous line). Dotted line gives the Rayleigh distribution (exact to the first order). Data are obtained from numerical simulations.

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