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ROGUE WAVES IN OCEANIC TURBULENCE

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ABSTRACT

A stochastic model of wave groups is presented to explain the occurrence of exceptionally large waves, usually referred to as rogue waves. The model leads to the description of the non-Gaussian statistics of large waves in oceanic turbulence and to a new asymptotic distribution of their crest heights in a form that generalizes the Tayfun model. The new model explains the unusually large crests observed in flume experiments of narrowband waves. However, comparisons with realistic oceanic measurements gathered in the North Sea during an intense storm indicate that the generalized model do not appear to improve upon the original Tayfun distribution.

Keywords: rogue wave; stochastic wave group; wave turbulence; probability of exceedance; Gaussian sea; quasi-determinism, Slepian model.

1 INTRODUCTION

Rogue waves are extreme events with potentially devastating effects on offshore structures and ships. A rogue wave event occurred on January 1^{st} 1995 at the Draupner platform in the North Sea provides evidence that such waves can occur in the open ocean. Theoretical models attempt to explain various physical mechanisms that can produce such a focussing of wave energy in a small area of the ocean. When nonlinearities are negligible, ocean waves are usually modeled by the linear superimposition of a large number of elementary waves having amplitudes related to a given spectrum and random phases. In this case, large waves are due to the linear focussing of a wave group (Lindgren 1972, Boccotti 2000) and their crests and troughs are both Rayleigh-distributed. If second-order nonlinearities are dominant, then the sea surface displays sharper narrower crests and shallower more rounded troughs. As a result, the skewness of surface elevations is positive (Longuet-Higgins 1963), and wave crests are distributed according to the Tayfun model (Tayfun 1980). If, however, elementary waves also exchange energy nonlinearly via third order four-wave resonances, narrowband wave trains can undergo intense modulational instability enhancing the occurrence of large waves (Fedele 2008, Tayfun & Fedele 2007, Janssen 2003) and the distribution of crest heights deviates from the Tayfun model. This is confirmed by the wave-flume experiments in (Onorato et al. 2006) and the numerical simulations of the Dysthe equation (Socquet-Juglard et al. 2005), a special case of the Zakharov equation (Zakharov 1999). However, in broadband waves the Tayfun distribution appears to explain crest statistics well (Fedele 2008, Tayfun & Fedele 2007, Onorato et al. 2005, Socquet-Juglard et al. 2005). The unusually wave crests observed in both the latter experiments and simulations are well explained by a recent Gram-Charlier approximation of the crest distribution proposed in (Tayfun & Fedele 2007), which is based on heuristic arguments. The derivation of this crest model stems from the general Hermite expansion of random variables, and it does relate to the physics of nonlinear waves only through the statistical estimations of both the skewness and kurtosis of the wave surface. Could such type of Gram-Charlier crest models be derived directly from the basic equations governing the ocean dynamics, without any use of Hermite-type expansions ?

An answer to this question is attempted in this paper. The main contribution of this work is the formulation of a new model of stochastic wave groups which provides a theoretical framework for the non-Gaussian statistics of large waves in oceanic turbulence. This is defined as the chaotic behavior of a sea of weakly nonlinear coupled dispersive wave trains in evolution according to the Zakharov equation. Stochastic wave groups describe the dynamics of the wave surface around a randomly chosen large crest (Lindgren 1972, Boccotti 2000), and their nonlinear space-time evolution reveals the statistical structure of large wave crests and thus their expected shape. A generalization of the Tayfun model for the statistical distribution of crest heights over large waves is then derived. Finally, comparisons with the experimental data of (Onorato et al. 2006), numerical simulations of (Socquet-Juglard et al. 2005) and data collected in the North Sea are presented.

2 OCEANIC TURBULENCE

Consider weakly nonlinear random waves propagating in water of uniform depth d. The sea surface displacement ζ from the mean sea level is given by

$$\zeta\left(\mathbf{x},t\right)=\zeta_1\left(\mathbf{x},t\right)+\zeta_2\left(\mathbf{x},t\right), \tag{1}$$

where

$$\zeta_1\left(\mathbf{x},t\right)=\int b(\mathbf{k},t)e^{i\boldsymbol{\theta}}d\mathbf{k}+c.c. \tag{2}$$

is the first order component with $\boldsymbol{\theta}=\mathbf{k}\cdot\mathbf{x}-\omega t$, b a time-varying complex coefficient and

$$\zeta_2\left(\mathbf{x},t\right)=\frac{1}{4}\int b_1 b_2\left[A_{12}^{+}e^{i(\boldsymbol{\theta}_1+\boldsymbol{\theta}_2)}+A_{12}^{-}e^{i(\boldsymbol{\theta}_1-\boldsymbol{\theta}_2)}\right]d\mathbf{k}_{1,2}+c.c.$$

represents the second-order correction; for brevity $b_1=b(\mathbf{k}_1,t)$, $\boldsymbol{\theta}_1=\mathbf{k}_1\cdot\mathbf{x}-\omega_1 t$, $A_{12}^{\pm}=A^{\pm}(\mathbf{k}_1,\mathbf{k}_2)$ are interaction coefficients (see e.g. Forristall 2000), $\mathbf{k}$=horizontal wave-number vector, with $k=|\mathbf{k}|$, $\mathbf{x}=(x,y)$ is the horizontal position vector, ω is the angular frequency related to k via $gk\tanh kd=\omega^2$. Further, the complex amplitude b_1 varies in time according to the Zakharov system (Zakharov 1999)

$$i\frac{\partial b_1}{\partial t}=2\epsilon\int gW_{34}^{12}\sqrt{\frac{\omega_1}{\omega_2\omega_3\omega_4}}\bar{b}_2 b_3 b_4\delta_{34}^{12}\exp\left(i\omega_{34}^{12}t\right)d\mathbf{k}_{2,3,4}$$

where W_{34}^{12} is the interaction kernel, $\omega_{34}^{12}=\omega_1+\omega_2-\omega_3-\omega_4$, $\delta_{34}^{12}=\delta\left(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3-\mathbf{k}_4\right)$, and ϵ a characteristic wave steepness. Drawing upon (Choi et al. 2005), the perturbation expansion of b_1 in small ϵ is given, correct to $O(\epsilon)$, by

$$b_1(t)=C_1(t)(1+i\Omega_1 t)-2\epsilon\int gW_{34}^{12}\cdot \tag{3}$$
$$\cdot\sqrt{\frac{\omega_1}{\omega_2\omega_3\omega_4}}\bar{C}_2 C_3 C_4\delta_{34}^{12}\frac{\exp\left(-i\tilde{\omega}_{34}^{12}t\right)-1}{\tilde{\omega}_{34}^{12}}d\mathbf{k}_{2,3,4},$$

and the long-term behavior of the surface ζ is known up to times of $O(1/\epsilon)$. Here,

$$C_1(t)=A_1\exp\left(i\Omega_1 t\right),\quad \Omega_1=2\epsilon\omega_1\int W_{12}^{12}\left|A_2\right|^2 d\mathbf{k}_1$$

with $C_1(t)=C_1(\mathbf{k}_1,t)$, $A_1=A(\mathbf{k}_1)$, Ω_1 as the renormalization frequency arising from the nonlinear frequency shift due to self-interactions and $\tilde{\omega}_{34}^{12}=\omega_{34}^{12}+\Omega_1+\Omega_2-\Omega_3-\Omega_4$.

If nonlinear effects are neglected, i.e. $\epsilon=0$, then the linear surface displacement ζ_1 is an ergodic, stationary Gaussian process. The mean frequency ω_m and bandwidth ν of the wave spectral density $S\left(\mathbf{k}\right)$ of ζ_1 irrespective of direction are given, respectively, as

$$\omega_m=\frac{m_1}{m_0},\quad \nu=\sqrt{\frac{m_0 m_2}{m_1^2}-1}, \tag{4}$$

where $m_j=\int\omega^j S\left(\mathbf{k}\right)d\mathbf{k}$ is the jth order spectral moment. The space-time covariance Ψ of ζ_1 is given by

$$\Psi(\mathbf{X},T)=\int S\left(\mathbf{k}\right)\cos(\mathbf{k}\cdot\mathbf{x}-\omega T)d\mathbf{k}, \tag{5}$$

and $\psi(T)=\Psi(\mathbf{0},T)$ is the time covariance for brevity. It is assumed that the first absolute minimum of $\psi(T)$ occurs at time $T=T^*$ and that $\psi(T)$ decreases monotonically between $T=0$ when the absolute maximum is attained and $T=T^*$. Clearly $\langle\zeta_1\rangle=0$, and the variance of ζ_1 is given by $\left\langle\zeta_1^2\right\rangle=\Psi(\mathbf{0},0)=m_0=\sigma^2$, where $\langle\cdot\rangle$ stands for expected value. The small steepness parameter $\epsilon=\mu_m^2$, where $\mu_m=\sigma\omega_m^2/g$.

3 LARGE CRESTS IN GAUSSIAN SEAS

Assume that a large wave crest of amplitude h is recorded at $\mathbf{x}=\mathbf{x}_0$ and $t=t_0$ during a sea storm. What happened in the space-time neighborhood of $(\mathbf{x}_0,t_0)$ when the large crest is observed ? Boccotti (2000) showed that as $h/\sigma\to\infty$, with probability approaching 1, the large crest occurs when a well defined wave group ζ_c, in transit through $\mathbf{x}_0$, reaches its apex. The large crest h is also the largest crest height of ζ_c. The surface displacement of ζ_c around $\mathbf{x}=\mathbf{x}_0+\mathbf{X}$ and $t=t_0+T$ is asymptotically described by the sum of a deterministic part $\zeta_{\rm det}$ of $O(h)$ and a residual random process R_ζ of $O(1)$ (Lindgren 1972, Boccotti 2000), viz.

$$\zeta_c(\mathbf{X},T)=\zeta_{\rm det}(\mathbf{X},T)+R_\zeta(\mathbf{X},T), \tag{6}$$

where

$$\zeta_{\det}(\mathbf{X},T) = \langle \zeta_1(\mathbf{X},T)\,|\zeta_1(\mathbf{0},0)=h\,\rangle = \frac{h}{\sigma^2}\Psi(\mathbf{X},T).$$

Thus, ζ_c represents the conditional process $\zeta_1(\mathbf{X},T)\,|\zeta_1(\mathbf{0},0)=h$ and $\zeta_{\det}$ is its conditional expectation. As $h/\sigma \to \infty$, the residual R_ζ in (6) becomes negligible relative to the first term, and a high local maximum also corresponds to a local wave crest since $\zeta_{\det}$ attains its absolute maximum at $(T=0, \mathbf{X}=\mathbf{0})$. Moreover, ζ_c can also be interpreted as the wave surface around a randomly chosen large crest (Lindgren 1972, Boccotti 1989, 2000) if h is assumed to be a random variable described by the Rayleigh probability density. Thus, $\zeta_c \approx \zeta_{\det}$ is asymptotically correct to $O(h)$, and either represents the wave surface locally to a given crest height h, or it defines the conditional process of the wave surface around a randomly chosen crest if h is Rayleigh-distributed.

4 STOCHASTIC WAVE GROUPS

Physically, ζ_c represents a wave group in which the largest crest occurs as waves, growing from the tail of the group, reach its apex and then decay at the front of the group (Boccotti 1989,2000). This section is devoted to the asymptotic characterization of the $O(h^0)$-random residual R_ζ of (6) in terms of the space-time covariance Ψ. This shall lead to an improved expression of the wave group ζ_c, where the random residual R_ζ is explicitly determined. First, the wave profile $\eta_c(T)$ at $\mathbf{X}=\mathbf{0}$ is expressed in terms of an $O(h)$ contribution $\eta_{\det}(T) = \zeta_{\det}(\mathbf{0},T)$ and the random residual $r(T) = R_\zeta(\mathbf{0},T)$ of $O(h^0)$ as

$$\eta_c(T) = \eta_{\det}(T) + r(T) \tag{7}$$

where $\eta_{\det}(T) = \zeta_{\det}(\mathbf{0},T) = h\psi(T)/\sigma^2$. Drawing upon (1), the effects of the residual $r(T)$ on η_c are now determined. Specifically, as $h/\sigma \to \infty$, with probability approaching 1, the surface profile locally near a large crest tends to assume the shape given by $\eta_{\det}(T)$ (Lindgren 1972, Boccotti 2000). The latter represents a wave profile with a crest of amplitude h at time $T=0$ followed by a local minimum of amplitude $\eta_{\det}(T^*)$ at $T=T^*$, with T^* being the abscissa of the first local minimum of $\psi(T)$ (see point P in figure 1). Further, when the absolute minimum of $\psi(T)$ occurs at $T=T^*$, then $\eta_{\det}(T)$ represents a large wave with period $T_h \approx 2T^*$ and a crest-to-trough amplitude given by $h\left(1-\psi(T^*)/\sigma^2\right)$. For large values of h, the wave trough of the profile $\eta_c(T)$ following the crest of amplitude h shall now occur at time $T = T^* + u$, shown as point $P\prime$ in figure 1, with u being random. As $h/\sigma \to \infty$, a crest of amplitude h that occurs at $T=0$, is followed after a time lag T^*+u by a trough, and $\eta_c(T)$ and its first time derivative $\dot{\eta}_c(T)$ at $T=T^*$ attain values given, correct to $O\left(h^0\right)$, by

$$\eta_c(T^*) = \eta_{\det}(T^*) + \Delta + O\left(h^{-1}\right), \tag{8}$$

$$\dot{\eta}_c(T^*) = -\ddot{\eta}_{\det}(T^*)u + O\left(h^{-1}\right).$$

Conversely, if the conditions in (8) hold, then a crest of amplitude h at time $T=0$ is followed by a trough at time $T = T^*+u$. For linear Gaussian functions, an approximation to $\eta_c(T)$ satisfying both sets of the preceding conditions exactly is given by

$$\eta_c(T) = \eta_{\det}(T) + \frac{\Delta}{\sigma^2}\frac{-\psi^*\psi(T) + \psi(T-T^*)}{1-\psi^{*2}}, \tag{9}$$

where u drops out ignoring terms of $O(h^{-1})$, and $\psi^* \equiv \psi(T^*)/\psi(0)$. With the random residual r of $O(1)$ explicitly determined now, it can be differentiated from $\eta_{\det}(T)$ of $O(h)$ in (9). It is straightforward to extend the above time formulation to the space-time domain obtaining a new approximation of the wave group ζ_c in (6) in the form

$$\zeta_c(\mathbf{X},T) = \zeta_{\det}(\mathbf{X},T) + \frac{\Delta}{\sigma^2}\frac{-\psi^*\ \Psi(\mathbf{X},T) + \Psi(\mathbf{X},T-T^*)}{1-\psi^{*2}}. \tag{10}$$

Evidently, this is an improved expression of the wave surface locally around a large crest, where the random residual R_ζ in (6) is explicitly determined as $\Delta/h \to 0$, and terms of $O(h^{-1})$ have been neglected. Note that from (10), for a given h, averaging over Δ yields the conditional mean $\zeta_{\det}$ as expected.

For a given h, ζ_c is the conditional process locally around a given crest, i.e. $\zeta_1(\mathbf{X},T)\,|\zeta_1(\mathbf{0},0)=h$. Instead, if h and Δ are interpreted as random, then ζ_c identifies a *stochastic wave group*, describing the dynamics locally around a randomly chosen crest. As $h/\sigma \to \infty$, the dimensionless variables $\xi = h/\sigma$ and $\tilde{\Delta} = \Delta/\sigma$ are stochastically independent. Moreover, ξ is Rayleigh-distributed and $\tilde{\Delta}$ is Gaussian with zero mean and variance equal to $1-\psi^{*2}$.

5 NONLINEAR STOCHASTIC GROUPS AND LARGE CRESTS

Consider now waves in oceanic turbulence. The linear structure of the surface ζ_1 is distorted by second order nonlinearities and third order four-wave resonances, to yields

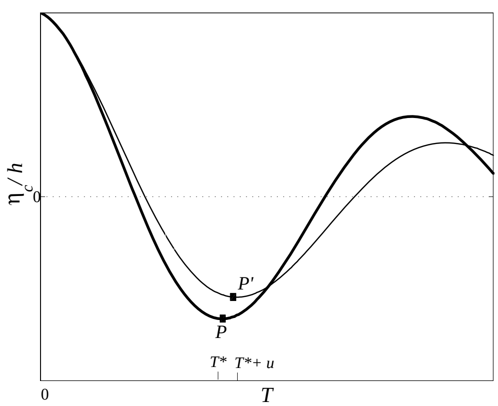

Figure 1. Wave profile η_c locally around a large crest.

the nonlinear surface ζ. Moreover, the crest statistics deviates from the Rayleigh conditions. Drawing upon (Fedele 2006,2008 and Fedele & Arena 2005), the non-Gaussianity of the wave crest heights can be quantified by investigating the structure of ζ locally around a large crest of amplitude h_{nl} recorded at $\mathbf{x} = \mathbf{x}_0 + \mathbf{X}$ and $t = t_0 + T$ during a sea storm. One can still ask what happened in the space-time neighborhood of $(\mathbf{x}_0, t_0)$ when the large crest is observed. The answer to this query is given by the following nonlinear conditional process

$$\zeta_{nc} = \left(\zeta(\mathbf{X}, T) \,|\zeta(\mathbf{0}, 0) = h_{nl}\right).$$

In general ζ_1 assumes some amplitude value h at $(\mathbf{X} = \mathbf{0}, T = 0)$, and this implies that

$$\zeta_{nc} = \left(\zeta(\mathbf{X}, T) \,|\zeta_1(\mathbf{0}, 0) = h\right),$$

but the relationship between h and h_{nl} is unknown, and generally ζ_1 does not attain a crest at $(\mathbf{X} = \mathbf{0}, T = 0)$ if ζ_{nc} does. However, stationary points of ζ_1 corresponding to large maxima, are also the critical points of large maxima of ζ, correct to $O(\epsilon)$, because waves are weakly nonlinear (Fedele 2008). In weakly nonlinear waves, if a large crest of the nonlinear ζ occurs with amplitude h_{nl}, then most likely the linear ζ_1 attains also a crest with an amplitude h. Thus, the occurrence of a large crest of ζ is due to the weakly nonlinear evolution of ζ_c, that is

$$\zeta_{nc} = \left(\zeta \,|\zeta_1 = \zeta_c\right) = f(\zeta_c)$$

where ζ_c is the Gaussian group in (10), and $f(\zeta_1)$ is the nonlinear mapping between ζ_1 and ζ known from both (1) and the perturbation expansion (3). In physical terms, ζ_{nc} is a nonlinear group that, prior to focussing, tends to reflect the characteristics of the Gaussian group ζ_c. As $\xi \to \infty$, $f(\zeta_c)$ is obtained by setting the linear component ζ_1 in (1) equal to $\zeta_c\left(\mathbf{x} - \mathbf{x}_0, t - t_0\right)$.

The amplitude h_{nl} of the largest crest of ζ_{nc} occurs approximately at $\mathbf{x} = \mathbf{x}_0$ and $t = t_0$ and neglecting $O(\tilde{\Delta}^3)$ terms it is a function of the linear h via the dimensionless equation

$$\xi_{\max} = \xi + \frac{\mu}{2}\xi^2 + \mathcal{I}(t_0)\ \xi^3 + \mathcal{A}(t_0)\xi^2\tilde{\Delta} + \mathcal{B}(t_0)\xi\tilde{\Delta}^2, \quad (11)$$

where $\xi_{\max} = h_{nl}/\sigma$, and $\mu = \lambda_3/3$ is related to the skewness coefficient $\lambda_3 = \left\langle \zeta^3 \right\rangle/\sigma^3$ of the wave surface. Moreover $\mathcal{I}$, $\mathcal{A}$ and $\mathcal{B}$ are multidimensional integrals in $(\mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$ space, viz.

$$\mathcal{I} = \mathcal{P}\int Q_{34}^{12} S_2 S_3 S_4 d\mathbf{k}_{2,3,4},$$
$$\mathcal{A} = \mathcal{P}\int Q_{34}^{12} S_2 S_3 S_4 \left(g_1 + g_2 + g_3\right) d\mathbf{k}_{2,3,4},$$
$$\mathcal{B} = \mathcal{P}\int Q_{34}^{12} S_2 S_3 S_4 \left(\bar{g}_1 g_2 + \bar{g}_1 g_3 + g_2 g_3\right) d\mathbf{k}_{2,3,4},$$

where $\mathcal{P}$ means principal value, and

$$Q_{34}^{12} = \frac{1}{2}\frac{g\epsilon}{m_0^2} W_{34}^{12} \sqrt{\frac{\omega_1}{\omega_2\omega_3\omega_4}} \delta_{34}^{12} \frac{1 - \cos\left(\omega_{34}^{12} t_0\right)}{\omega_{34}^{12}}.$$

where $\mathcal{P}$ means principal value. Drawing upon Janssen (2003), the coefficient $\mathcal{I}$ relates to the fourth-order cumulant $\lambda_{40} = \mu_4 - 3$ of the wave surface as $\lambda_{40} = 24\mathcal{I}$, μ_4 being the kurtosis. In the narrowband limit, as $\nu \to 0$, $\mathcal{A} \approx O(\nu)$ and $\mathcal{B} = -3\mathcal{I}/\left(1 - \psi^{*2}\right) + O(\nu)$. As $\xi \to \infty$, ignoring terms of $O(\tilde{\Delta}^3)$ and averaging over $\tilde{\Delta}$ yield the probability of exceedance for the nonlinear wave crest height as

$$\Pr\left\{\xi_{\max} > \xi\right\} = \exp\left(-\frac{1}{2}\xi_0^2\right)\left(1 + \frac{\lambda_{40}}{24}\ \xi^2\left(\xi^2 - 3\right)\right) \quad (12)$$

where $\xi = \xi_0 + \frac{\mu}{2}\xi_0^2$. We shall refer to this asymptotic result, as the generalized Tayfun (GT) distribution, which is similar to the Gram-Charlier (GC) approximation proposed in (14), viz.

$$P_{GC}\left(\xi\right) = \exp\left(-\frac{1}{2}\xi_0^2\right)\left(1 + \frac{\lambda_{40}}{24}\ \xi^2\left(\xi^2 - 4\right)\right). \quad (13)$$

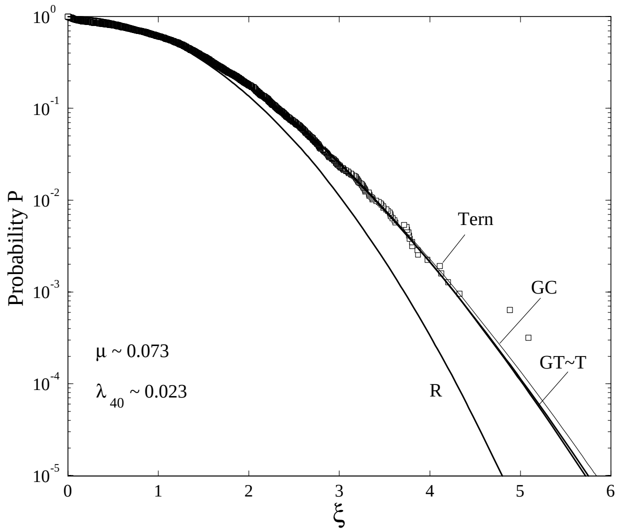

Figure 2. Crest exceedances from Tern in comparison with the Tayfun, generalized Tayfun and Gram-Charlier models. Labels: R=Rayleigh, T = Tayfun (μ), GT= generalized Tayfun (μ, λ_{40}), GC= Gram-Charlier.

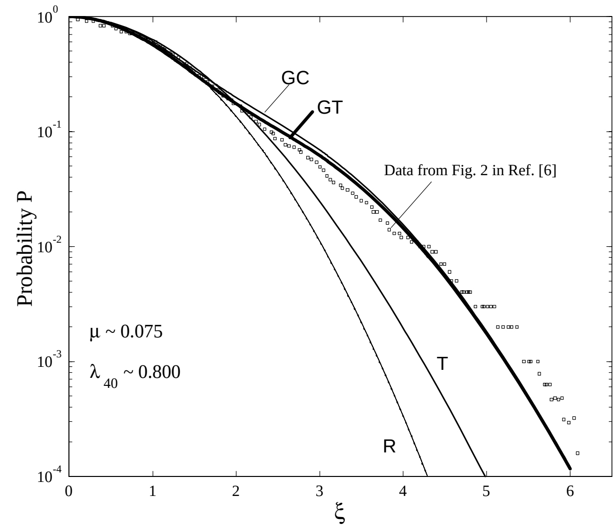

Figure 3. Crest height distribution from wave-flume experiments (figure 2 in Onorato et al. 2006) in comparison with the Tayfun, generalized Tayfun and Gram-Charlier models. Labels are as for figure 2.

Note that, the GC model in Tayfun & Fedele (2007) is based on the Hermite expansion of random variables and it does not lead to an explicit expression for λ_{40} in terms of spectral properties of ζ as it does in the case of the GT model, which is instead based on the physics of the Zakharov equation.

For directional broad-band waves, wave number quadruplets are in perfect resonance, i.e. $\omega_{34}^{12} = 0$, and the Tayfun (T) model (Tayfun 1980) is recovered in (12) since $\lambda_{40} = 0$. Deviations from the Tayfun distribution can occur in long-crested narrowband waves due to modulational instability (Janssen 2003). The expected shape of large waves is given by the the nonlinear conditional expectation $\langle \zeta \,|\zeta_1 = \zeta_c \rangle = \langle \zeta(\mathbf{X}, T) \,|\zeta_1(\mathbf{0}, 0) = h \rangle$ that follows from averaging over $\tilde{\Delta}$ as

$$\langle \zeta \,|\zeta_1 = \zeta_c \rangle = f(\zeta_{\text{det}}) + O\left(\sigma_c^2\right), \tag{14}$$

where σ_c^2 the linear conditional variance of ζ_c.

6 COMPARISONS

Consider the data set which comprises 9 hours of measurements gathered during a severe storm in January, 1993 with a Marex radar from the Tern platform located in the northern North Sea in 167 m water depth. This data set is hereafter simply referred to as Tern. Tern represents storm seas under fairly steady conditions with broadband spectra characterized with $\sigma = 3.024\ m$, spectral bandwidth $\nu = 0.629$ and observed $\lambda_3 = 0.174$. A stable estimate of the steepness μ in terms of spectral properties is given by $\mu_a = \mu_m \left(1 - \nu + \nu^2\right)$. In figure 2, the empirical distribution from Tern is compared with the T ($\mu \simeq \mu_a = 0.073$), GT ($\mu \simeq \mu_a = 0.073$, $\lambda_{40} \simeq 0.023$) and GC models respectively. It is observed that both the GT and GC models doe not appear to improve significantly the predictions derived from the simpler T model. For most practical applications, the differences between the models appear insignificant, falling within a band of 1-2 %. Consider now the case of unidirectional narrowband waves. The trend of the experimental wave-flume data of figure 2 in (Onorato et al. 2006) is reproduced and shown in figure 3 together with the predictions based on GT, GC ($\mu \simeq 0.075$, $\lambda_{40} \simeq 0.80$) and T ($\mu \simeq 0.075$) models. The original T model tends to underestimate the data whereas both the GT and GC models appear to explain data qualitatively well. The latter models also describe well the crest height distribution from figure 9 (case C) of Socquet-Juglard et al. (2005) obtained from numerical simulations of the Dysthe equation. This is reproduced and shown in figure 4 in comparison with the GT, GC ($\mu \simeq 0.07$, $\lambda_{40} \simeq 0.40$) and T ($\mu \simeq 0.07$) models.

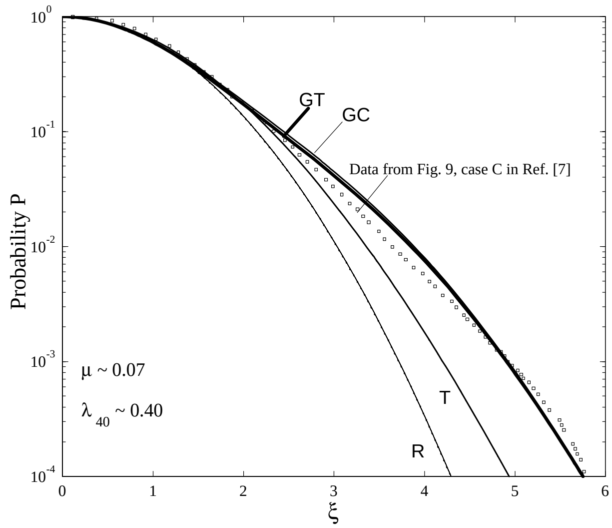

Figure 4. Crest exceedances from numerical simulations (figure 9, case C in Socquet-Juglard et al. 2005) in comparison with the Tayfun, generalized Tayfun and Gram-Charlier models. Labels are as for figure 2.

7 CONCLUSIONS

A new theoretical framework for the non-Gaussian statistics of large waves in oceanic turbulence is proposed based on the concept of stochastic wave group. A generalized Tayfun model for the statistical distribution of crest heights over large waves is then presented. The new crest model can explain the large deviations from the Tayfun distribution observed in flume experiments of narrowband waves, but for realistic oceanic sea states its improvement on the predictions of the Tayfun model appear to be insignificant.

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