Proceedings of The Thirteenth (2003) International Offshore and Polar Engineering Conference
Honolulu, Hawaii, USA, May 25−30, 2003

ISBN 1−880653-60−5 (Set); ISSN 1098−6189 (Set)

On the Statistics of High Non-linear Random Waves

Francesco Fedele
Department of Civil Engineering - University of Vermont
Burlington, Vermont, USA

Felice Arena
Department of Mechanics and Materials - University 'Mediterranea' of Reggio Calabria
Loc. Feo di Vito, Reggio Calabria, Italy

ABSTRACT

It is well known that in a Gaussian sea state for an infinitely narrow spectrum the crest and the trough heights follow the Rayleigh distribution, because of linearity of the first order Stokes expansion solution (Longuet-Higgins 1952). For spectra of finite bandwidth Boccotti (1981, 2000) obtained, as a corollary of his first formulation of the theory of quasi-determinism (which is exact to the first order in a Stokes expansion), that the crest (trough) height still follows asymptotically the Rayleigh law for high waves in a Gaussian sea states.

In this paper we extend the theory of quasi-determinism of Boccotti to the second-order, deriving a new wave-crest distribution that takes in account non-linear effects and is valid for finite-bandwidth of the spectrum. The theoretical predictions are finally compared with data of non-linear numerical simulation.

KEY WORDS: crest height; second-order effect; probability of exceedance; finite bandwidth; gaussian sea state; quasi-determinism; wave group.

INTRODUCTION

The first order Stokes solution of the free surface displacement is a random Gaussian process of time. Longuet-Higgins (1952) showed that for an infinitely narrow spectrum the wave height follows the Rayleigh distribution. Because of the symmetry of a gaussian sea state the crest and trough heights follow the same Rayleigh law for narrow spectra. For spectra of finite bandwidth, Boccotti (1981, 2000) showed as corollary of his first formulation of the theory of quasi-determinism that the crest height still follows asymptotically a Rayleigh law for high waves in gaussian sea states [see also Lindgreen (1970,1972), Leadbetter & Rootzen (1988), Kac & Slepian (1959), Sun (1993), Breitung (1996), Maes & Breitung (1997)]. If the non-linear effects are not negligible, higher crests are more probable than higher troughs and the free surface displacement tends to deviate from being gaussian (Longuet-Higgins, 1963). The latter crest-trough asymmetry was investigated by Tayfun (1980) and Tung & Huang (1985); they derived a probability distribution of the second order crest (trough) height under the hypothesis of narrow-band spectrum. A more general second-order narrow-band model was proposed by Arena & Fedele (2002); they obtained the crest and the trough distributions of a general non-linear stochastic family, which includes many processes in the mechanics of the sea waves (either in an undisturbed field or in front of a vertical wall).

Two models (Prevosto et al., 2000; Forristall, 2000) have been proposed for the crest height distribution of three dimensional waves: they give results very close to each other and in good agreement with field data (Prevosto & Forristall, 2002).

In this paper we extend the work of Boccotti (1982, 1983, 2000) deriving a new wave-crest distribution that takes in account second-order effects and is valid for finite-bandwidth of the spectrum. In his work, Boccotti showed that in a Gaussian sea state, if it is known that a very high local maximum (very high with respect to the mean crest height) occurs in some time and location, this implies that a well defined quasi-deterministic wave group generates the highest local maximum which tends to be the crest of its wave. As corollary he derived that the probability of exceedance of the crest (Boccotti, 1981, 1989, 2000) follows asymptotically the Rayleigh distribution.

The authors, starting from the general second order Stokes solution of the surface displacement, show how the amplitude of the non-linear crest depends upon the linear crest amplitude. Thus the probability distribution of the non-linear crest is obtained.

Comparison between theoretical and numerical simulation distributions is finally proposed.

NON-LINEAR EFFECTS FOR FINITE-BANDWIDTH OF THE SPECTRUM

The theory of quasi-determinism

We shall derive an asymptotic formula for the non-linear wave crest distribution, based on the theory of quasi-determinism by Boccotti (1982, 1989, 1993, 1997, 2000). He showed that, if in a Gaussian sea state it is known that a very high local maximum occurs in some location and time, this implies with high probability that a well defined wave-group generates the high local maximum (see also Lindgreen (1970,1972)). In detail if a local wave maximum of given elevation h_o occurs at a time t_o at a fixed point x_o, y_o, with probability approaching 1, the surface displacement at x_o+X, y_o+Y is asymptotically equal to the deterministic form

$$\bar{\eta}_L(x_o+X, y_o+Y, t_o+T) = \frac{\Psi(X,Y,T;x_o,y_o)}{\Psi(0,0,0;x_o,y_o)} h_0 \tag{1}$$

if $h_0/\sigma \to \infty$, i.e. the crest is very high with respect to the mean crest height, being σ the standard deviation of the free surface displacements. The space-time covariance $\Psi(X,Y,T;x_o,y_o)$ is defined as

$$\Psi(X,Y,T;x_o,y_o) \equiv < \eta(x_o,y_o,t)\eta(x_o+X,y_o+Y,t+T) >, \tag{2}$$

where

$$< f(t) >= \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau f(t)dt \tag{3}$$

is the ensemble average. An exceptionally high local maximum, with a very high degree of probability, is also a wave crest of its wave, because the absolute maximum of the autocovariance function $\Psi(X,Y,T)$ is at $X=0, Y=0, T=0$. A direct consequence is that the number of wave crests exceeding a fixed threshold b tends to coincide with the number of local wave maxima exceeding it, provided the fixed threshold is very high; which in its turns implies: the number of wave crests exceeding a very high threshold b tends to coincide with the number of b up-crossings (b_+); that is

$$\frac{N_{cr}(b;\Delta t)}{N_+(b;\Delta t)} \to 1 \quad \text{as} \quad b/\sigma \to \infty, \tag{4}$$

where $N_{cr}(b;\Delta t)$ and $N_+(b;\Delta t)$ denote respectively the number of wave crests exceeding the threshold b and the number of b_+ in the very large time interval Δt; it follows that

$$P(h_0 > b) = \exp\left(-\frac{b^2}{2\sigma^2}\right) \quad \text{as} \quad b/\sigma \to \infty. \tag{5}$$

As regard to the free surface for long-crested random waves in deep water from Eq. (1), we derive the surface displacement at x_o+X when an exceptional crest height of given elevation h_0 occurs at a time t_o at a fixed point x_o:

$$\bar{\eta}_L(x_o+X, t_o+T) = \frac{h_0}{\sigma^2}\int_0^\infty S(\omega)\cos(kX-\omega T)\, d\omega \tag{6}$$

where $S(\omega)$ is the wave spectrum. Let us note that $\int_0^\infty S(\omega)d\omega = \sigma^2$.

Non-linear free surface displacement in deep water for a given initial local maximum

For long-crested waves in deep water, the general second order solution for the surface displacements is (Sharma & Dean 1979; Tayfun 1986)

$$\eta(x,t) = \sum_n a_n \cos\psi_n + \frac{1}{4}\sum_{n,m} a_n a_m \cdot \\ \cdot\left[(k_n+k_m)\cos(\psi_n+\psi_m) - |k_n-k_m|\cos(\psi_n-\psi_m)\right] \tag{7}$$

in which $\psi_n = k_n(x_0+X) - \omega_n T + \vartheta_n$ and the coefficients $\{a_n\}_{n\in\aleph}$, $\{\vartheta_n\}_{n\in\aleph}$ are undetermined.

Let us assume that the free surface displacement has a local maximum h at point $x = x_o$ and that this maximum occurs at time $t = t_o$. We shall derive an expression of the free surface displacement $\bar{\eta}(X,T)$ at point $x_o + X$ at time instant $t_o + T$, when h is very large with respect to the standard deviation of the free surface displacement.

The conditions of a stationary point at time $t = t_o$ (that is T=0) at the location point $x = x_o$ (that is $X = 0$) are

$$\bar{\eta}(X,T) \text{ such that } \begin{cases} \bar{\eta}\,|_{X=0,T=0} = h \\ (\partial\bar{\eta}/\partial x)|_{X=0,T=0} = 0 \end{cases} \tag{8}$$

The solution of problem (8) (see Appendix) has expression as

$$\eta(X,T) = \frac{h_0}{\sigma^2}\int_0^\infty S(\omega)\cos\psi\, d\omega + \\ + \frac{h_0^2}{4\sigma^4}\int_0^\infty\int_0^\infty S(\omega_1)S(\omega_2)\left[(k_1+k_2)\cos(\psi_1+\psi_2) + \right. \\ \left. -|k_1-k_2|\cos(\psi_1-\psi_2)\right]d\omega_1 d\omega_2 \tag{9}$$

where $\psi = kX - \omega T$ and it is valid for high crest height, i.e. for $h_0/\sigma \to \infty$. The initial high wave crest is

$$h = h_0 + \frac{h_0^2}{4g\sigma^4}\int_0^\infty\int_0^\infty S(\omega_1)S(\omega_2)\left[(\omega_1^2+\omega_2^2) - |\omega_1^2-\omega_2^2|\right]d\omega_1 d\omega_2. \tag{10}$$

Formula (10) is valid as long as the non-linear effects are weak, which means that $k_p h_0$ has to be smaller, where k_p is a wave number corresponding to a characteristic frequency ω_p. For wind wave spectra, in deep water, k_p can be assumed equal to ω_p^2/g, being ω_p the peak frequency. If $h_0/\sigma \to \infty$, the non-linear effects are negligible, i.e. $k_p h_0 \to 0$, if the steepness $\varepsilon_p = k_p\sigma$ goes to zero as

$\left(h_0/\sigma\right)^{-1-\kappa}\quad \kappa>0$.

By setting the change of variable $w_1=\omega_1/\omega_p$, $w_2=\omega_2/\omega_p$ and by defining the non-dimensional spectrum $\tilde{S}(w)=\omega_p\,S(\omega_p w)/\sigma^2$ Eq. (10) is rewritten in the form

$$h=h_0+\alpha(S)\frac{h_0^2}{\sigma}$$

$$\alpha(S)=\frac{\varepsilon_p}{4}\int_0^\infty\int_0^\infty \tilde{S}(w_1)\tilde{S}(w_2)\left[\left(w_1^2+w_2^2\right)-\left|w_1^2-w_2^2\right|\right]dw_1dw_2. \tag{11}$$

The variance of the second order process is easily derived from (11) and has expression as

$$\sigma_\eta^2=\frac{\sigma^2}{\beta^2} \tag{12}$$

$$\beta=\frac{1}{\sqrt{1+\frac{\varepsilon_p^2}{2}\int_0^\infty\int_0^\infty \tilde{S}(w_1)\tilde{S}(w_2)\left(w_1^4+w_2^4\right)dw_1dw_2}}. \tag{13}$$

So we have that the non-dimensional wave crest height $\xi_{high}=h/\sigma_\eta$ can be expressed as the following

$$\xi_{high}=\beta\,u+\alpha(S)\beta\,u^2 \tag{14}$$

where the random variable $u=h_0/\sigma$ has Rayleigh distribution. As consequence the probability of exceedance of the absolute maximum (crest) is:

$$P\left(\xi_{high}>\xi\right)=\exp\left[-\frac{1}{8\alpha^2}\left(1-\sqrt{1+\frac{4|\alpha|\xi}{\beta}}\right)^2\right] \tag{15}$$

valid for $\xi\to\infty$.

Conditions to have a wave profile

The quasi-deterministic structure defined by Eq. (1) is a wave group as long as at the location X=0 the time profile is a wave. Following Boccotti (2000) we shall assume that the autocovariance function, defined as

$$\Psi(T)=\int_0^\infty S(\omega)\cos(\omega T)d\omega\,, \tag{16}$$

is monotonic decreasing in [0,T*], where T* is the first absolute minimum of $\Psi(T)$. This implies that the linear solution $\eta_L(X=0,T)$ has a local maximum at T=0 and a local minimum at T=T* which are respectively the crest and the trough of a wave. For weakly non-linear effects the latter abscissas T=0 and T=T* are also respectively the abscissas of local maximum and local minimum of the non-linear surface $\eta(X=0,T)$ as long as the two following conditions hold

$$\left.\frac{\partial^2\eta}{\partial T^2}\right|_{X=0,T=0}<0\qquad \left.\frac{\partial^2\eta}{\partial T^2}\right|_{X=0,T=T^*}>0 \tag{17}$$

which are sufficient conditions to have respectively a local maximum at T=0 and a local minimum at T= T*. The first condition in (17) gives the following inequality

$$\frac{h_0}{\sigma^2}\int_0^\infty \tilde{S}(w)w^2dw+$$

$$\frac{h_0^2}{4\sigma^4}\int_0^\infty\int_0^\infty \tilde{S}(w_1)\tilde{S}(w_2)\left[\left(w_1^2+w_2^2\right)(w_1+w_2)^2+\right. \tag{18}$$

$$\left.-\left|w_1^2-w_2^2\right|(w_1-w_2)^2\right]dw_1dw_2>0$$

which is satisfied with probability 1 for any spectrum and wave height h_0 (Let us observe that dimensionless spectrum $\tilde{S}(w)\geq 0\ \forall w$ and $\left(w_1^2+w_2^2\right)(w_1+w_2)^2-\left|w_1^2-w_2^2\right|(w_1-w_2)^2>0$).

The second condition in (17) imposes the following inequality instead

$$h_0+\varepsilon_p\tau\frac{h_0^2}{\sigma}>0 \tag{19}$$

where

$$\tau(S)=\left\{\int_0^\infty\int_0^\infty \tilde{S}(w_1)\tilde{S}(w_2)\left[\left(w_1^2+w_2^2\right)(w_1+w_2)^2\cos\left(w_1+w_2\right)\omega_pT^*+\right.\right.$$

$$\left.\left.-\left|w_1^2-w_2^2\right|(w_1-w_2)^2\cos\left(w_1-w_2\right)\omega_pT^*\right]dw_1dw_2\right\}\Big/ \tag{20}$$

$$\Big/\left(4\int_0^\infty \tilde{S}(w)w^2\cos w\omega_pT^*dw\right)$$

Let us observe that for typical wind wave spectra $\pi/2<\omega_pT^*<\pi$ and $\tau(S)<0$. In this case condition (19) is not satisfied if

$$\frac{h_0}{\sigma}>\frac{1}{\varepsilon_p|\tau(S)|}. \tag{21}$$

Because $u=h_0/\sigma$ is a random variable with Rayleigh distribution, the probability failure can be evaluated as

$$P(S)=P\left[u>\frac{1}{\varepsilon_p|\tau(S)|}\right]=\exp\left[-\frac{1}{2\varepsilon_p^2\tau^2(S)}\right]. \tag{22}$$

P(S) can be interpreted as the fraction of the realizations of the non-linear stochastic process in which there is no local minimum at T=T*. This probability failure is less or equal to an assigned value P if the steepness ε_p is less than the threshold $\left(\varepsilon_p\right)_{\lim}$ defined as

$$\left(\varepsilon_p\right)_{\lim} = \frac{1}{|\tau(S)|}\sqrt{\frac{1}{2\ln(1/P)}}\,. \tag{23}$$

For the case of narrow band spectrum $\left(\varepsilon_p\right)_{\lim}$ is equal to 0.1345 for P=1/1000 in agreement with Arena & Fedele (2002). As the spectrum gets broader the latter upper bound reduces in order to avoid distortion of the non-linear wave profile. Indeed small steepness is required to have a non-linear profile smoothly varying like the linear profile.

The rectangular spectrum

In this section we shall specialize to the case of rectangular spectra. The following spectral form is considered

$$\tilde{S}(w) = \frac{1}{w_{\max} - w_{\min}}, \qquad w_{\min} \le w \le w_{\max} \tag{24}$$

with $1 \ge w_{\min} \ge 0$,$w_{\max} \ge 1$ (where the dimensionless frequency $w \equiv \omega / \omega_p$, being now ω_p the mean frequency $\omega_m \equiv (\omega_{\min} + \omega_{\max})/2$). The parameter α can be evaluated explicitly by solving analytically the double integral in eq. (11), obtaining the following expression

$$\alpha(w_{\min}, w_{\max}) = \varepsilon_p \frac{(w_{\min} + w_{\max})^2 + 2w_{\min}^2}{12}\,. \tag{25}$$

Let us observe first that for narrow-band spectrum

$$\alpha_\infty = \lim_{w_{\min}\to 1,\, w_{\max}\to 1} \alpha = \varepsilon_p / 2 \tag{26}$$

in agreement with the narrow-band probability of exceedance (see, for example, Arena & Fedele 2002).

APPLICATION

In order to validate formula (15) for the probability of exceedance of the non-linear crest we have used Monte Carlo method: by using eq. (7) to generate the realizations of a non-Gaussian sea state, we have performed four simulations (30000 waves for each simulation). In figure 1-2 the plots of the theoretical curves evaluated using the analytical expression (7) are compared to the probabilities of exceedance derived from the Monte Carlo simulations along with the corresponding narrow-band curves, centered at the mean frequency ω_m. The spectral parameters used are $w_{\min} = 0.75, w_{\max} = 1.25$ with two different values of the steepness ε_p (0.055 and 0.021).

In fig. 3-4 other plots relative to spectra with parameters $w_{\min} = 0.50, w_{\max} = 1.50$ and ε_p equal to either 0.055 or 0.021.

The probabilities derived from the simulation agree well with the analytical probabilities. Moreover the non-linear effects for the finite-band spectrum are smaller than for narrow-band case, which means that $\alpha < \alpha_\infty$.

CONCLUSIONS

A new analytical expression for the probability of exceedance of crest in a non-Gaussian sea state has been derived based on the theory of quasi-determinism of Boccotti (1981-2000). The new proposed formula considers second order non-linearities due to the finite bandwidth of the spectrum. Monte Carlo simulations of sea states with rectangular spectra have been performed and they agree well with the analytical probabilities.

APPENDIX

In order to solve problem (8) let us apply a perturbation approach. The assigned height h is expanded as

$$h = h_0 + h_1 + h_2 + \ldots\ldots\ldots\ldots \tag{A1}$$

where h_0, h_1, h_2, … are unknown parameters to be determined. We assume that $h_0 \propto \sigma$, $h_1 \propto \sigma^2$, …, $h_n \propto \sigma^{n+1}$,..., where σ is the standard deviation of the surface displacement. From the general solution (7) the two following equations are derived

$$h_0 + h_1 + h_2 + \ldots\ldots\ldots = \sum_{n=1}^{N} a_n \cos\vartheta_n + \frac{1}{4}\sum_{n=1}^{N}\sum_{m=1}^{N} a_n a_m \cdot \left[(k_n + k_m)\cos(\vartheta_n + \vartheta_m) - |k_n - k_m|\cos(\vartheta_n - \vartheta_m)\right] \tag{A2}$$

$$0 = -\sum_{n=1}^{N} a_n k_n \sin\vartheta_n + \frac{1}{4}\sum_{n=1}^{N}\sum_{m=1}^{N} a_n a_m \cdot \left[-(k_n + k_m)^2 \sin(\vartheta_n + \vartheta_m) + |k_n - k_m|(k_n - k_m)\sin(\vartheta_n - \vartheta_m)\right] \tag{A3}$$

where $\vartheta_n = k_n x_0 - \omega_n t_0 + \varepsilon_n$. Because we assume $a_n \propto \sigma$, only the first two terms in the h expansion are non zero. All the terms higher than the second order vanish. To the first order, Equations (A2-A3) give respectively

$$O(\sigma) \qquad h_0 = \sum_{n=1}^{N} a_n \cos\vartheta_n \qquad 0 = \sum_{n=1}^{N} a_n k_n \sin\vartheta_n \tag{A4}$$

The second equation in (A4) is satisfied if $\vartheta_n = 0 \;\; \forall n$ whatever are the values of the coefficients $\{a_n\}_{n\in\aleph}$. The latter solution is not unique. There exist other solutions with non zero phases ϑ_n.

Physically the condition $\vartheta_n = 0 \;\; \forall n$ can be interpreted as linear focusing. In fact the first equation gives

$$h_0 = \sum_n a_n \tag{A5}$$

which is the highest value that h_0 can reach for an assigned spectrum. Therefore the condition $\vartheta_n = 0 \;\; \forall n$ implies that an absolute maximum is reached at a fixed point $x = x_o$ at time instant $t = t_o$ by the first order solution. From Boccotti's theory if a very large crest height h_0 occurs at a fixed point $x = x_o$ at time instant $t = t_o$, the free surface displacement [Eq. (6)] in discrete form is given by

$$\bar{\eta}_L(X,T) = \sum_{n=1}^{N} \tilde{a}_n \cos\tilde{\psi}_n \tag{A6}$$

where

$$\tilde{a}_n = \frac{h_0}{\sigma^2} S(\omega_n) \mathrm{d}\omega \tag{A7}$$

and

$$\tilde{\psi}_n = k_n X - \omega_n T \,. \tag{A8}$$

Because the high wave group at $(X = 0, T = 0)$ attains a maximum it follows

$$\bar{\eta}_L (X = 0, T = 0) = h_0 \quad \Rightarrow \sum_{n=1}^{N} \tilde{a}_n = h_0 \tag{A9}$$

and

$$(\partial \bar{\eta}_L / \partial X)_{X=0,T=0} = 0 \quad \Rightarrow \sum_{n=1}^{N} \tilde{a}_n \sin \tilde{\psi}_n = 0 \tag{A10}$$

(let us observe that $\tilde{\psi}_n = 0 \;\; \forall n$ at *X*=0, *T*=0). The last two equations (A9) and (A10) are identical to the equations (A4) if $\tilde{\psi}_n = \psi_n$, $\tilde{a}_n = a_n$, which implies

$$\vartheta_n = 0 \;\; \forall n \;\; \text{and} \;\; a_n = \frac{h_0}{\sigma^2} S(\omega_n) \mathrm{d}\omega \tag{A11}$$

To the second order, Equations (A2-A3) give

$$O(\sigma^2) \begin{cases} h_1 = \dfrac{1}{4} \displaystyle\sum_{n,m} a_n a_m \left[(k_n + k_m) \cos(\vartheta_n + \vartheta_m) + \right. \\ \left. - |k_n - k_m| \cos(\vartheta_n - \vartheta_m) \right] \\ \\ 0 = \dfrac{1}{4} \displaystyle\sum_{n,m} a_n a_m \left[- (k_n + k_m)^2 \sin(\vartheta_n + \vartheta_m) + \right. \\ \left. + |k_n - k_m| (k_n - k_m) \sin(\vartheta_n - \vartheta_m) \right] \end{cases} \tag{A12}$$

because $\vartheta_n = 0 \;\; \forall n$, the second equation is satisfied while the first condition becomes of the form

$$h_1 = \frac{1}{4} \sum_{n,m} a_n a_m \left[(k_n + k_m) - |k_n - k_m| \right]. \tag{A13}$$

By considering Eq. (A11), that is $a_n = h_0 / \sigma^2 S(\omega_n) \mathrm{d}\omega$, we obtain, in continuous form

$$h_1 = \frac{h_0^2}{4\sigma^4} \int_0^\infty \int_0^\infty S(\omega_1) S(\omega_2) \left[(k_1 + k_2) - |k_1 - k_2| \right] \mathrm{d}\omega_1 \mathrm{d}\omega_2 \,. \tag{A14}$$

Finally, we have that, if a very large crest height occurs, the second order height may be written as:

$$h = h_0 + \frac{h_0^2}{4\sigma^4} \int_0^\infty \int_0^\infty S(\omega_1) S(\omega_2) \cdot \left[(k_1 + k_2) - |k_1 - k_2| \right] \mathrm{d}\omega_1 \mathrm{d}\omega_2 + o(\sigma^2) \tag{A15}$$

More in general, the second order free surface displacement, when a very high crest occurs at time instant t_o at point x_o, is given by:

$$\bar{\eta}(X,T) = \frac{h_0}{\sigma^2} \int_0^\infty S(\omega) \cos\psi \, \mathrm{d}\omega + \frac{h_0^2}{4\sigma^4} \int_0^\infty \int_0^\infty S(\omega_1) S(\omega_2) \cdot \left[(k_1 + k_2) \cos(\psi_1 + \psi_2) - |k_1 - k_2| \cos(\psi_1 - \psi_2) \right] \mathrm{d}\omega_1 \mathrm{d}\omega_2 \,. \tag{A16}$$

REFERENCES

Arena, F, and Fedele, F (2002) "A family of narrow-band non-linear stochastic processes for the mechanics of sea waves ". *Eur. J. Mech. - B/Fluids*, Vol 21(1), pp 125-137.

Boccotti, P (1981). "On the highest waves in a stationary Gaussian process". *Atti Acc. Ligure di Scienze e Lettere*, Vol 38, pp 271-302.;

Boccotti, P (1982). "On the highest sea waves". *Monograph Institute of Hydraulics*, University of Genoa, 1-161.

Boccotti, P (1982). "On ocean waves with high crests". *Meccanica*, Vol 17, pp 16-19.

Boccotti, P (1983). "Some new results on statistical properties of wind waves". *Applied Ocean Research*, Vol 5, pp 134-140.

Boccotti, P (1989). On mechanics of irregular gravity waves. *Atti Acc. Naz. Lincei, Memorie*, Vol VIII, pp 111-170.

Boccotti, P (1997) "A general theory of three-dimensional wave groups", *Ocean Engng*., Vol 24, pp 265-300.

Boccotti, P, Barbaro G, and Mannino L (1993). "A field experiment on the mechanics of irregular gravity waves". *J. Fluid Mech.*, Vol 252, pp 173-186.

Boccotti, P (2000). "Wave mechanics for ocean engineering", Elsevier Science, pp 1-496.

Breitung, KW (1996) " Higher order Approximations for maxima of random fields", Advances in nonlinear stochastic mechanics , Solid Mech. Appl., Vol 47, pp 79-88.

Forristall, GZ (2000). "Wave Crest Distributions: Observations and Second-Order Theory", *Journal of Physical Oceanography*, Vol 30, No. 8, pp 1931-1943.

Kac M, Slepian D, (1959) "Large excursions of Gaussian processes", Ann. Math. Statist., Vol 30, n 4 pp 1215-1228.

Leadbetter, MR, Rootzen, H (1988) "Extremal Theory for Stochastic Processes", The annals of Probability, Vol 16, 2, pp 431-478.

Lindgren, G (1970), "Some properties of a normal process near a local maximum", Ann. Math. Statist. Vol. 41, n. 6 pp. 1870-1883.

Lindgren, G (1972), "Local maxima of Gaussian fields" Ark. Mat. Vol 10, pp 195-218.

Longuet-Higgins, MS (1963) "The effects of non-linearities on statistical distributions in the theory of sea waves", *J. Fluid Mech.* Vol 17, pp 459-480.

Longuet-Higgins, MS (1952). "On the statistical distribution of the heights of sea waves", *J. Mar. Res*. Vol 11, pp 245-266.

Maes, M.A, Breitung, KW (1997) "Direct Approximation of the extreme value distribution of nonhomogeneous Gaussian random fields", J. Offshore Mech. Arctic. Engng. Vol 119 pp 252-256.

Prevosto, M, Krogstad, HE and Robin, A (2000). "Probability distributions for maximum wave and crest heights". *Coastal Engng.*, Vol 40, pp 329-360.

Prevosto, M., Forristall GZ (2002) "Statistics of wave crests from models vs. measurements", Proc. 21st International Conference on Offshore Mechanics and Arctic Engineering (OMAE 2002), Oslo, OMAE 28443 paper.

Sharma, J.N. and Dean, R.G., 1979, Development and Evaluation of a Procedure for Simulating a Random Directional Second Order Sea Surface and Associated Wave Forces, Ocean Engineering Report No.20, University of Delaware.

Sun, J (1993) "Tail probabilities of the maxima of gaussian random fields", The Annals of Probability vol. 21, no. 1, pp. 34-71.

Tayfun, MA (1980). "Narrow-band nonlinear sea waves", *J. Geophys. Res*. Vol 85, pp 1548-1552.

Tayfun, MA (1986). " On Narrow-Band Representation of Ocean Waves ", *J. Geophys. Res.* Vol 91, pp 7743-7752.

Tung, CC, Huang, NE (1985) "Peak and trough distributions of nonlinear waves", *Ocean Engng.* Vol 12, pp 201-209.

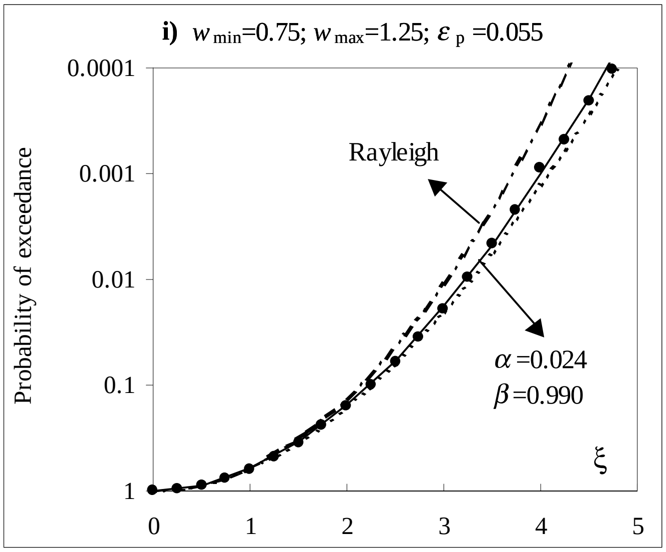

Figure 1. The crest height distributions. Comparison between Rayleigh law (broken line), narrow-band second-order distribution (dotted line), finite-band second-order distribution (Eq. 22 – continuous line) and data by numerical simulation.

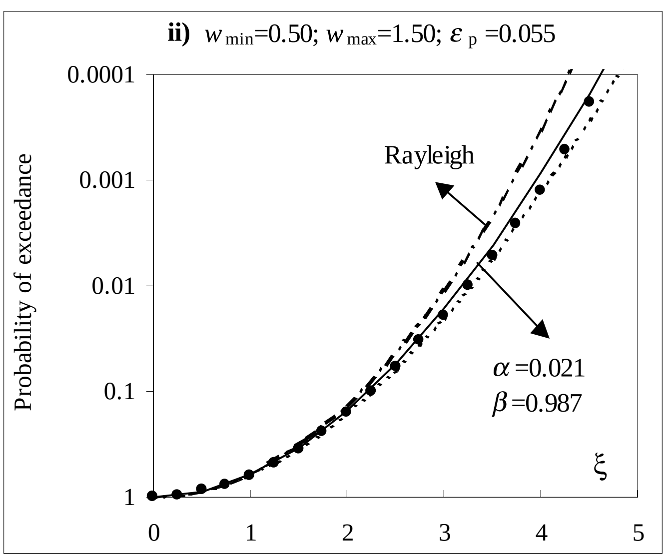

Figure 2. See caption of Figure 1.

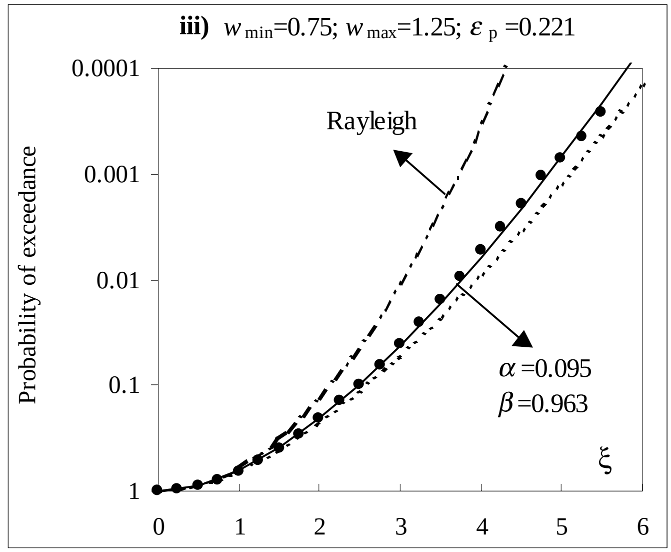

Figure 3. See caption of Figure 1.

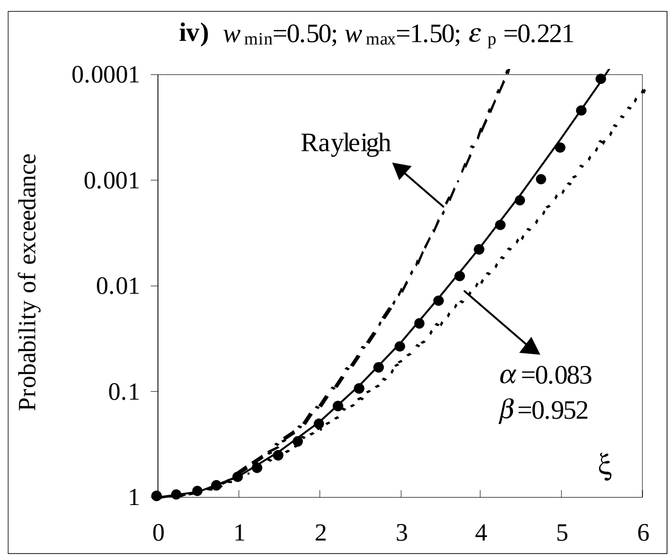

Figure 4. See caption of Figure 1.